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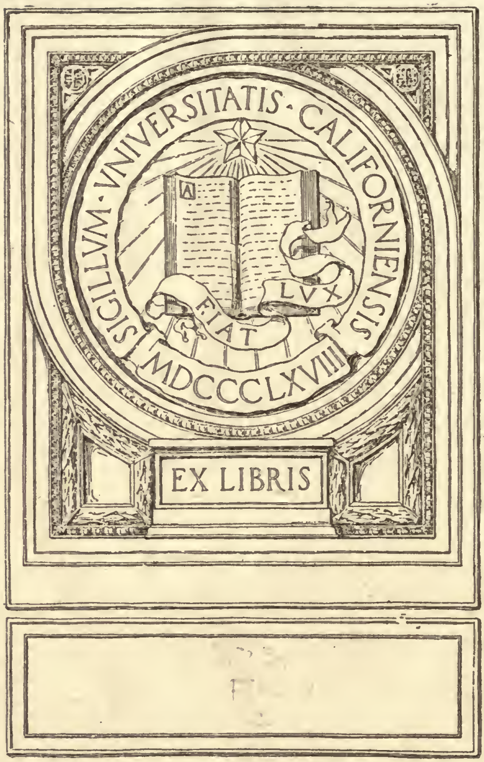
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A CALENDAR OF LEADING EXPERIMENTS

Bacon long ago listed in his quaint way the things which seemed to him most needful for the advancement of learning, and among other things he mentioned A Calendar of Leading Experiments for the better Interpretation of Nature.

A CALENDAR OF LEADING EXPERIMENTS

BY

WM. S. FRANKLIN AND BARRY MACNUTT

UNIV. OF CALIFORNIA

SOUTH BETHLEHEM, PA.
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The authors are teachers, and they consider teaching to be the greatest of fun, but they never yet have been helped in their work by anything they have ever read concerning their profession.

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PREFACE.

(*a*) "A boatman sits on a seat, braces his feet against a cleat and pulls on an oar. What forces act on the boatman's body?" The earth pulls on the boatman, the seat pushes on the boatman, the cleat pushes on the boatman and the oar pulls on the boatman. This is all very simple to one who has acquired the habit of analytical thinking, but a large group of sophomore engineering students got an average of 45 per cent in their answers to the question after two weeks of insistent coaching *on the fundamental notion of force action*, and nearly every human aspect of boating was represented in the answers, including even the chance of a ducking, for several of the young men would have it that the *water* pushes on the boatman's body.

(*b*) Ask a student of elementary mechanics *how a body* (*a particle*) *behaves when it is acted upon by an unbalanced force*, and the natural habit of thinking-in-terms-of-human-values shows itself after the most exacting class-room drill. Instead of giving his attention narrowly to *what is taking place at an instant*, the student, in his groping for human values (which do not exist in the bare elements of a subject), is pretty sure to refer to past history and to future prospects; his idea as to what *is taking place* is apt to remain widely inclusive, as if he were thinking of a complex human experience like two hours at a play, or a month's vacation, or four years of war. Very often the homely wording of the question as stated above betrays the student into a disclosure of his native contempt for precise ideas and he answers naïvely that "the body moves in the direction of the force." It is very difficult to develop the habit of analytical thinking.

Our teachers of mathematics seem to know that they are responsible for a certain kind of training, and they are not ashamed to apply themselves narrowly in its accomplishment.

Indeed, many of us *who do not share their responsibility* are inclined to think that mathematics teachers apply themselves too narrowly, because their students, as we get them, are deficient in mathematical ideas although strong, it maybe, on forms.

Physics teachers also have a definite responsibility, and our notion as to this responsibility is suggested by examples *a* and *b* above. The physics teacher, however, is strongly imbued with the idea that his function is to disclose to the student his (the physicist's) "domain of nature," and he is inclined therefore to discuss results—Helmholtz's theory of the origin of the sun's heat, Lord Kelvin's calculations of the age of the earth, and many other things of equal speculative interest—but such things prematurely considered divert the student's attention away from elementary or irreducible ideas and conceptions without which no analytical thinking is possible.

Results are fine inventions
For gentlemen who see;
But the micro-scope is needed
In this emergency.

Primarily this book has to do with class-room experiments in physics. The best experiments are those that are homely and simple, and suggestive rather than informing. The physics lecturer should pull ideas out of things like a prestidigitateur, and many of his demonstrations should be mere motion experiments,* by which we do not mean experiments which have to do with the theory of motion! We do not believe in the lecture experiment which aims at a numerical result rather than at an idea,—decidedly we do not; and our pet aversion is the "study in still life," if we may so describe the deadly lantern slide. Imagine a sleight-of-hand performer making use of lantern slides! It is unthinkable, and yet there never was a sleight-of-hand performer who had one tenth of the resources of the physics lecturer in college. In a two-years' course in elementary physics

* See pages 51 and 82 for examples.

we use, by actual count, 31 lantern slides; and out of a total of 48 lecture periods in the two years' course we have always used 8 or more for written tests on text-book and recitation work. This may seem to some as a damning confession, but the constraint which leads to analytical thinking cannot be made to operate through any combination of talk and show. No Missourian was ever regenerated by talk; and even when you have shown him he remains what he was before—a Missourian.

Secondarily this book is intended to set forth the possibilities of an extended course in elementary dynamics, including the dynamics of wave motion.

Some things in this book may seem to call for apology, for we simply cannot refrain from an occasional diversion in the way of poking fun, because so many things in teaching *are* funny, from our point of view.

From our point of view. Let no one who reads this book lose sight of the qualification. We might, of course, have dwelt here and there on faults of our own, but such things are not funny —from our point of view. For example, we have many times caught ourselves mistaking the fixity of an idea for its *raison dêtre*, whereas the fixity of an idea is not the same thing as reason, especially when it comes to making the idea clear to a young student. Also we have many times been victims of the "illusion of activity" by which we mean the sense of one's effectiveness which comes from being wholly engaged in an undertaking. "The lecture we gave this morning; how complete and perfect it was; what a masterpiece of edification!" The only cure for this illusion is the reaction of the students, and no teacher who seeks and uses this cure can be proud of his work; although he may develop a humble but invincible self-respect in that he never fails to do his best.

W. S. Franklin,
Barry MacNutt,

October 27, 1917.

TABLE OF CONTENTS.

DISCONNECTED ESSAYS.

UNIV. OF CALIFORNIA

PART I.

MECHANICS.

A very simple and complete treatment of the elementary theory of elasticity is given on pages 182 to 218 of Franklin and MacNutt's *Mechanics and Heat,* The Macmillan Co., New York City, 1910; and some fundamental experimental demonstrations in elasticity are described by Franklin and MacNutt on pages 90–100 of the *Bulletin of the Society for the Promotion of Engineering Education*, October, 1916.

THE STUDY OF SCIENCE.

Everyone knows of the constraint which is placed upon men by the physical necessities of the world in which we live, and although in one way this constraint is more and more relieved with the advancement of the sciences, in another way it becomes more and more exacting. It is indeed easier to cross the Atlantic ocean now than it was in Lief Ericsson's time; but think of the discipline of the present day shop and consider the rules of machine design, the rigors of the mathematical sciences! Could even the hardy Norsemen have known anything as uncompromisingly exacting as these?

It is no wonder that easy-going believers in liberal education have always looked with horror on the sciences, very much as softened men and women look upon work. Liberalism means freedom, and "liberalism in education is the freedom of development in each individual of that character and personality which is his true nature." All this we accept in a spirit of optimism, believing men's true natures to be good; but there is a phase of education which has but little to do, directly, with character and personality, and we call to your attention this conception of liberalism in education in order that we may turn sharply away from it as an incomplete conception which to a great extent excludes the mathematical sciences. Indeed we wish to point out a condition in education which is the antithesis of freedom; for the study of the mathematical sciences means a reorganization of the work-a-day mind of a young man as complete in its sphere as the pupation of an insect, and an exacting constraint is the essential condition of this reorganization.

This statement is taken from The Study of Science, an introduction to Franklin and MacNutt's *Mechanics and Heat*, The Macmillan Co., 1910. This essay is reprinted in *Bill's School and Mine*, A Collection of Essays on Education, by W. S. Franklin, published by Franklin, MacNutt and Charles, South Bethlehem, Pa. In this reprint it is stated that the essay "is a sticker, and that any particular reader who does not like it can leave it alone." But there is an increasing number of young men in this world who must study science whether they like it or not; and this essay is intended to explain this remarkable and in many respects distressing fact.

MASS AND WEIGHT.

"In defense of accuracy we must be zealous, as it were, even to slaying."—P. G. Tait.

Many of our teachers of physics and engineering are lacking in complete precision of thought concerning elementary mechanics largely because they refuse to think of the fundamental ideas of mechanics with a greater degree of mathematical or logical precision than seems to them to be required by the degree of measuring precision that satisfies the research physicist and the engineer; but precision of thought is not dependent upon precision of measurement. Indeed precision of measurement in modern physics is mostly a tradition which we have inherited from spherical astronomy; precision of thought is vastly more important.

The term *mass* means quantity of material as measured by a balance scale, and it is properly expressed in grams or pounds. In commerce, however, we speak of the *weight* of a batch of sugar or coal as *weighed* by a balance scale, and the word *weight* so used means exactly the same thing as *mass*.

When we speak of the *weight* of a body in this text we mean *the force with which the earth pulls on the body*, and it is properly expressed in dynes or poundals. The pull of the earth on a one-pound body in London is a definite force, it is extensively used as a unit of force, and it is called the "pound." The weight of a body, that is to say, the earth-pull on a body may be properly expressed in "pounds" *but the coal man's scales do not determine the weight of a body in "pounds."*

One "pound" of force gives an acceleration of 32.174 feet per second per second to a mass of one pound, so that, according to Newton's second law, one "pound" of force will give an acceleration of one foot per second per second to 32.174 pounds

of material (one *slug* of material). Therefore if force is expressed in "pounds," mass in slugs and acceleration in feet per second per second the simple form of the equation $F = ma$ can be used.

To reduce pounds of coal as measured by a balance scale to slugs, divide by 32.174 (a pure number, *not* an acceleration). The weight W of a body (the force with which the earth pulls a body) is of course equal to mg, so that the mass of the body m equals W/g. Thus the actual local weight of a body *in London "pounds"* divided by the local value of g gives the mass of the body in slugs*; but when your coal man sends you a bill for what he calls a certain "weight" of coal, say 2,000 pounds, do not divide this kind of "weight" by g to get mass!

The system of mechanical units which involves the "pound" as a unit of force is very widely and properly used, and every teacher of elementary mechanics should use this system of units unhesitatingly. We call the system the *foot-slug-second system* for obvious reasons. There is no objection to the precise† use of the foot-slug-second system except on the ridiculous assumption that the primary business of the teacher is to promote the metric system; but it is improper for anyone to agree to use the word *weight* for the force with which the earth pulls on a body and then carelessly revert to the usage to the grocer and the coal man.

The weight of a body in London in "pounds" is, of course, equal to its mass in pounds, and it has, more than once, been proposed to call the weight of a body in London its *standard*

* A perplexing mix-up of dimensions is involved in the double meaning of the word pound. Out of respect for etymology we may postulate identical dimensions to the *pound* of sugar and the "*pound*" of push or pull, or we may postulate identical dimensions to pounds of sugar and slugs of sugar. We prefer the second postulate, and, in accordance with this postulate, one must divide pounds of sugar by 32.174 (a numeric) to get slugs of sugar, and one must divide the local weight of a body in London "pounds" by the local acceleration of gravity (a denominate number) to get mass in slugs. From this point of view the "pound" of force has the same dimensions as the poundal of force.

† See page 50 of this volume for further discussion of this subject.

weight and use the term *standard weight* instead of *mass;* but there are two objections to this, namely, (*a*) The term mass is almost universally recognized by physicists and chemists, and (*b*) The mass of a given body is independent* of time and place, it has to do only with an invariant* relation between the given body and the standard kilogram (a piece of metal), and extraneous and confusing ideas would be introduced by the use of the term *standard weight* because this term implies location and a relationship between the given body and the earth. How awkward it would be, for example, to be obliged always to speak and think of the distance d between two points (x, y, z) and (x', y', z') as $[(x - x')^2 + (y - y')^2 + (z - z')^2]^{1/2}$. This is an invariant and the most useful name or symbol for it is a name or symbol which carries no redundant suggestions as to particular axes of reference, *and this would be true even if we had always to make use of a particular set of reference axes in the measurement of d.*

Many engineering writers pretend never to use the poundal as a unit of force, and we use the poundal chiefly as a name for a step in an argument. It is better to name it than to refrain from naming it, thereby pretending not to use it. Everyone who expresses mass in pounds does use the poundal as a unit of force, and every English-speaking person expresses mass in pounds when he buys sugar or coal. If a man steals sugar no accounting is necessary, at leas no book-keeping is necessary, and only such men among English-speaking people can honestly claim not to use the poundal as a unit of force! The question would appear therefore to be a question of honesty versus dishonesty, but it is not quite as bad as that, for the second horn of the dilemma is unconscious dishonesty or sophistry, and this unconscious dishonesty has been carried to the limit by a recent writer who speaks of the coal man's unit of mass as scholastic.†

* No consideration is here given to variations of mass as recognized in the recent developments of the principle of relativity.

† Professor E. V. Huntington, of Harvard University. See *American Mathematical Monthly*, January, 1917, page 16. In his recent articles on the fundamentals

of dynamics Professor Huntington has not contributed anything to the logical or mathematical aspects of the subject which has not been known in all simplicity and completeness since the days of Thomson and Tait, he is definitely astray logically in his attempt to ignore the fundamental idea of mass (see page 10 of this volume), and he shows a peculiar lack of appreciation of the constraining exactions which are placed upon the physicist by measuring operations. For example, Professor Huntington says, with easy assurance that, "fundamental units may be chosen at pleasure," whereas the choice of fundamental units is *governed* by practical considerations; in the first place the fundamental units must be preserved as material standards, and in the second place *the fundamental quantities must be susceptible of very accurate measurement because the definition of a derived unit cannot be realized with greater accuracy than the fundamental quantities can be measured.*

Think of the years of confusion in electrical measurements when the theoretical ohm could not be produced with greater accuracy than one or two per cent and when anybody could make resistance measurements to one tenth of a per cent! When we think of this old nightmare and read Professor Huntington's argument for the adoption of force as a fundamental quantity, what do you suppose we are inclined to say? We refrain from saying it, at any rate. Professor Huntington assures us that a spring can be easily preserved, whereas the best and most carefully "aged" measuring springs in existence grow perceptibly softer and softer in use. Tempered steel is in a meta-stable condition and so also is hardened phosphor bronze and fused quartz. Of course the "pound" of force, the pull of the earth on the standard pound in London, is adequately preserved, but Professor Huntington seems nevertheless to think that a unit of force must be preserved in the form of a stretched spring!

One of the very few redeeming features of the recent, long-drawn-out rehash of elementary dynamics in *Science* and in the *American Mathematical Monthly* is the article by Professor L. M. Hoskins of Stanford University, *Science*, April 23, 1915, pages 608–611. The rehash is due primarily to Professor Huntington, and its significance lies almost solely in its showing that nearly everybody seems to take anybody seriously when anybody happens to be a Harvard professor. See page 34 of this volume for further comment.

ON UNBALANCED FORCE.

When a car is being started by a locomotive the forward pull of the locomotive on the car is greater than the backward pull or drag of track and wind on the car; all the forces which act on the car are together equivalent to a single force pulling forwards on the car, and this single force constitutes what is called an *unbalanced force*. Whenever the velocity of a body is changing the body is being acted upon by an unbalanced force. As an example let us consider a ball which is tied to a string and twirled in a circle. All the forces acting on the ball, namely, the gravity pull of the earth, the drag of the air and the pull of the string, are together equivalent to a single force.

One of the most frequent perplexities of the student in regard to unbalanced force comes from a wrong interpretation of the principle of equality of action and reaction. In the above example the string pulls inwards on the ball *and the ball pulls outwards on the string*. We refer here to the two forces acting at the point where the string is attached to the ball, and these two forces act on different bodies as stated, namely, the string pulls on the ball and the ball pulls on the string. Two such forces which act on different bodies have nothing whatever to do with the question as to *whether the forces which act on a given body are balanced or unbalanced*. This matter is discussed more at length on page 12 of *General Physics*.

The misunderstanding and false application of the principle of equality of action and reaction is so common that every teacher of physics should make a searching analysis of the matter; he should not accept anyone as authority but he should think for himself and he should by all means pay attention to simple essentials. Do you agree as to what is said above concerning unbalanced force? Do you agree that action and reaction are equal and opposite forces acting on different bodies and there-

fore having nothing whatever, directly or indirectly, to do with the question as to whether the forces which act on a *given body* are balanced or unbalanced? If you are doubtful consider a trade between two persons A and B. This trade is a purchase from A's point of view and a sale from B's point of view, and of course the sale aspect of this mutual operation is equal to its purchase aspect; but everyone understands that a given person may buy more than he sells or sell more than he buys. A trade is a *single thing*, and yet it is very convenient to speak of a trade as a sale or purchase in its relation to a given person. The force action between two bodies A and B, involving the exertion of equal and opposite forces on the two bodies, is a *single thing*, a *stress*, let us say, and yet it is convenient to speak of a stress as a force acting on A or as an equal and opposite force acting on B. Indeed it is logically necessary to think of one aspect of a stress when considering a given body. Iowa is north of Missouri—yes but Missouri is south of Iowa! He who would argue that an unbalanced force action on a given body cannot be because action and reaction are always exactly equal and opposite is like the purist who would wish to invent a fancy way of saying that Missouri is *south* of Iowa, fearing a verbal battle with the man from Missouri who knows that Iowa is *north* of Missouri! Imagine a man who, in considering that A is north of B and that therefore B is the same distance south of A, concludes that nothing can really be north or south of anything else. The geometric sense of such a man would be on a par with the mechanical sense of one who concludes that an unbalanced force cannot exist because action and reaction are always equal and opposite and on a par with the business sense of a merchant who would conclude from the equal and opposite aspects of a trade that it would be impossible to buy more than he sells!

All the forces which act on a given body are often equivalent to a single resultant unbalanced force, and it is such a force that changes the velocity of the body.

The recently developed electromagnetic theory (including the theory of relativity) does not vitiate this point of view in the least, although it does modify our notions of mass and point to minute (ordinarily extremely minute) force actions between accelerated bodies and the *ether* of space if we may use that term merely to designate widespread associated energy. Indeed the principle of relativity (for it should no longer be called a theory) does not vitiate the principle of equality of action and reaction if due attention be given to these etherial force actions.

A "body" includes everything inside of a certain bounding surface, and at moderate velocities and accelerations only an extremely minute fraction of the kinetic energy and momentum of a body resides outside of this boundary, according to the electromagnetic theory, and only an extremely minute fraction of an accelerating force is balanced by force action across this boundary;* and a recent attempt† to broaden the science of dynamics by postulating an ether force which completely balances every accelerating force is unjustifiable and indeed meaningless.

* The integral of the well-known Maxwellian stress across the bounding surface.

† Professor H. M. Dadourian's *Analytical Mechanics*. A very sane review of this book by Professor E. W. Rettger is given in *Science*, January 23, 1914, pages 140–142. See page 34 of this volume for further comment.

THE FUNDAMENTAL EQUATIONS OF DYNAMICS.

The recent attempt* to rule out mass as a fundamental concept in dynamics makes it advisable to supplement what is given in *General Physics* on the fundamental equations of dynamics (see pages 8–10). We are here concerned with the acceleration of a body when acted upon by an unbalanced force. The acceleration varies from body to body for a given identifiable force, and it varies from force to force for a given body. These are two equally fundamental modes of variation and both of them must be formulated in the fundamental equation or equations of dynamics, and of course this formulation must be based on experiment.

Given three bodies A, B and C, and three identifiable forces a, b and c. Let the acceleration of each body due to each force be observed, the results being as shown in the accompanying table.

TABLE OF OBSERVED ACCELERATIONS.

		Bodies		
		A	B	C
Forces	a	25	30	35
	b	50	60	70
	c	75	90	105

* By Professor E. V. Huntington who says that equation (2) below is a mathematical consequence of equation (1)! See *Science* March 3, 1916, page 315.

Equation (1) expresses the relation between the accelerations a and a' *of a given body* due to two forces F and F', respectively, and pure logic would not know of even the existence of any other body. See page 34 of this volume for further comment.

Let us suppose that the table has been extended so as to include a great many different forces and a great many different bodies, then a careful inspection of the table would lead to the following generalizations.

(*a*) If one force produces twice as much acceleration as another force when acting on a given body, then the one force produces twice as much acceleration as the other force when acting on *any body whatever.*

(*b*) If one body is accelerated twice as much as another body under the action of a given force, then the one body is accelerated twice as much as the other body under the action of *any force whatever.*

The experimental fact (*a*) makes it convenient to define the ratio of two forces as the ratio of the accelerations they produce when acting on a given body, because this ratio is the same for all bodies. That is

$$\frac{F}{F'} = \frac{a}{a'} \tag{1}$$

where a is the acceleration of a given body produced by force F, and a' is the acceleration produced by force F'.

The experimental fact (*b*) makes it convenient to define the ratio of the masses of two bodies as the inverse ratio of the accelerations produced by a given force, because this ratio is the same for all forces. That is

$$\frac{m}{m'} = \frac{a'}{a} \tag{2}$$

where a is the acceleration of body No. 1 and a' is the acceleration of body No. 2 both produced by a given force, and m and m' are the masses of the respective bodies.

Equations (1) and (2) are the fundamental equations of dynamics.

Experiment shows that the ratio m/m' as defined by equation (2) is exactly equal to the ratio of the masses of the two bodies as measured on a balance scale, and since the balance scale

measures mass very conveniently and with extreme precision it is best to *define* mass as measured by the balance scale and accept equation (2) as an experimental discovery.

From equations (1) and (2) it is evident that the acceleration of a body is proportional to the accelerating force and inversely proportional to the mass of the body.

Experiment 1.—The authors have found it best to use only extremely simple lecture experiments in the discussion of the laws of motion. Taking a box in the hands, call attention to the fact that you have to exert an upward force on the box to balance the downward pull of the earth. Call on a young man to pull on the box, and point out that in addition to supporting the box against gravity you have to balance the force exerted by the young man to keep the box stationary.

Point out that everything as stated applies to a box similarly held in a steadily moving car or boat on a straight track or course, so that the forces which act on a body are balanced when the body is stationary or moving uniformly along a straight path but surrounded on all sides by things moving along with it.

How about a body which moves steadily along a straight path but *not* surrounded by bodies which move along with it? Everyone knows that an active agent such as a horse or a steam engine must pull steadily on such a body to keep it in motion. If left to itself such a moving body quickly comes to rest. This tendency for a moving body to come to rest is due to *dragging forces* or *friction* exerted on the moving body by surrounding bodies. Thus a moving boat is brought to rest by the drag of the water when the propelling force ceases to act; a train of cars is brought to rest by a frictional drag when the pull of the locomotive ceases; a box which is drawn steadily across the table comes to rest when left to itself because of the dragging force due to friction between the moving box and the table. We must, therefore, always consider two forces when we think of a body which is kept in motion like a car or boat, namely, the *propelling force* due to some active agent and the *dragging force*

of friction. Newton pointed out that when a body is moving steadily along a straight path the propelling force is equal and opposite to the dragging force of friction. See *General Physics*, Art. 3, page 7.

Let us now consider the force* which must act on a body which is changing its velocity, upon a body which is being started or stopped, for example. Every one has noticed how a mule strains at his rope when starting a canal boat, especially if the boat is heavily loaded, and how the boat continues to move for a long time after the mule ceases to pull. In the first case the pull of the mule greatly exceeds the backward drag of the water, and the velocity of the boat increases; in the second case the backward drag of the water exceeds the pull of the mule, for the mule is not pulling at all, and the velocity of the boat decreases. When the velocity of a body is changing the forces which act on the body are unbalanced. We may therefore conclude that the effect of an unbalanced force on a body is to change the velocity of the body, and it is evident that the longer the unbalanced force continues to act the greater the change of velocity. Thus if the mule ceases to pull on a canal boat for one second the velocity of the boat will be but slightly reduced by the unbalanced backward drag of the water, whereas if the mule does not pull for ten seconds the velocity of the boat will be reduced to a much greater extent. *In fact the change of velocity of a body due to a given unbalanced force is proportional to the time that the force continues to act.* This is exemplified by a body falling freely under the action of the unbalanced pull of the earth on the body; during one second the body gains about 32 feet per second of velocity, during two seconds it gains twice as much velocity (about 64 feet per second), and so on.

Everyone knows what it means to give an easy pull or a hard pull on a body. That is to say, we all have an idea that a force may be large or small. Everyone knows also that under the

* If several forces act we here refer to the single force which is equivalent to them all. Translatory motion, only, is here considered.

action of a hard pull a canal boat will get under way more quickly (gain velocity faster) than it will under the action of an easy pull, *and a precise statement of the effect of an unbalanced force on a body must correlate the* **value of the force** *and* **the rate at which it imparts velocity to the body.** This seems a very difficult thing, but its difficulty is in large part due to the fact that we have not yet agreed as to what we mean when we say that one force is exactly three or four or any number of times as large as another force. Suppose therefore that *we agree to call one force twice as large as another when it will produce in a given body twice as much velocity in a given time* (remembering that we are talking about unbalanced forces). As a result of this definition we may state that *the amount of velocity produced per second in a given body by an unbalanced force is proportional to the force.*

Note.—This definition of the ratio of two forces is by no means sufficient to establish the equation $F/F' = a/a'$. A very wide range of experiment is necessary to show that this equation is true for all bodies if it is true for one. See discussion on pages 10 and 11 of this volume.

2. **An experiment with two similar blocks.**—Drop two similar blocks together and call attention to the fact that they reach

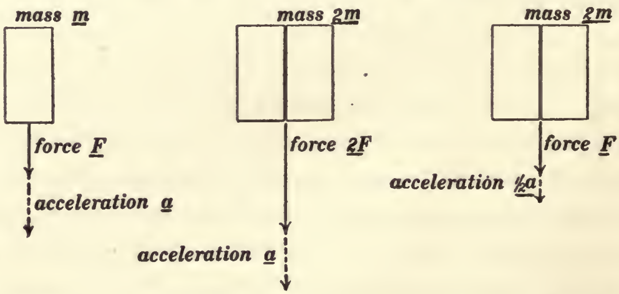

Fig. 1. Fig. 2. Fig. 3.

the floor at the same time. Two blocks fall the same distance in a given time, therefore they fall with the same increasing velocity and with the same acceleration as one block.*

* The friction of the air is very small and it is ignored.

The mass of the two blocks (as measured on a balance scale) is $2m$, where m is the mass of one block; and the pull of gravity on the two blocks is $2F$, where F is the pull gravity on one block. Therefore when m and F are both doubled the acceleration a remains unchanged.

This argument is not exactly in accordance with the fundamental equation (1) on page 11 of this volume. In all strictness it should be as follows: Let us adopt $F/F' = a/a'$ as the definition of the ratio of two forces, *and let us assume that the pull of the earth on the two blocks* (*mass* $2m$) *is twice as great as the pull of the earth on one block* (*mass* m), as indicated in Fig. 2. Imagine the force in Fig. 2 to be reduced to half-value; then according to the fundamental equation ($F/F' = a/a'$) the acceleration would be reduced to $\frac{1}{2}a$, as indicated in Fig. 3. Comparing Figs. 1 and 3 we see that when the mass of a body is doubled a given force F produces half as much acceleration, or $m/m' = a'/a$, which is equation (2) on page 11 of this volume. *Therefore the above assumption is justified*, that is, the weight of a body (pull of earth on the body) is proportional to the mass of the body as measured by a balance scale, and the ratio of two masses as defined by the equation $m/m' = a'/a$ is the same as the ratio of two masses as measured by a balance scale.

3. **Reaction experiment.**—An electrically driven toy locomotive runs on a circular track laid near the edge of a large horizontal disk which is supported on ball bearings so as to turn freely about a vertical axis. When the locomotive starts the disk is set turning backwards, and when the locomotive stops the disk is set turning forwards. An elevated railway structure has to be braced near a station so as to withstand the backward push of the locomotive as it starts a train and so as to withstand the very great forward push of an entire train which is brought to rest quickly by braking.

Remark.—Is the kick of a gun the *reaction* which corresponds to the push of the powder gases on the projectile? It is not. The kick of the gun is due to the backward push of the powder gases on the breech plug of the gun, and this is of course equal to the forward push of the breech plug on the powder gases (action is equal to reaction and opposite thereto). Also the forward push of the powder gases on the projectile is equal to the backward push of the projectile on the powder gases (action is equal to reaction and opposite thereto). But the forward push of the

breech plug on the powder gases is not equal to the backward push of the projectile on the powder gases; the powder gases are being rapidly accelerated and the forces which act on the powder gases are not balanced.

IMPULSE–VALUE OF A FORCE.

Consider an unbalanced force $F = ma$ acting on a body, and let us assume F to be constant. Multiply both members of this equation by the time t during which the force continues to act, and we have $Ft = mat$. But at (the rate of gain of velocity multiplied by elapsed time) is the total velocity v produced by the force F during the time t. Therefore we have:

$$Ft = mv$$

The product mv is called the *momentum* of the body, and the product Ft (the value of a force multiplied by the time it continues to act) is called the *impulse-value* of the force.

If the force F is not constant we have $F \cdot dt = m \frac{dv}{dt} \cdot dt = m \cdot dv$, and $\int F \cdot dt = \int m \cdot dv = mv$. The integral $\int F \cdot dt$ is called the impulse value of the force F and it is equal to the momentum mv which is produced.

The above equation applies not only to the starting of a body but also to the stopping of a body. For example, a hammer of mass m moving at velocity v strikes an obstacle and is brought to rest. Let F be the average force with which the obstacle pushes backwards against the hammer while stopping it (this is, of course, equal to the average force exerted by the hammer on the obstacle), and let t be the time during which this average force acts. Then $F = ma$, where F is the average backward force exerted on the hammer (or the average forward force exerted *by* the hammer) and a is the average rate at which the hammer is losing velocity. But during time t the hammer loses all of its velocity so that $at = v$, or $Ft = mat = mv$, where v is the initial velocity of the hammer (the total velocity lost by the hammer). Therefore the impulse-value of the force exerted by the hammer is equal to the momentum of the hammer. The force exerted by a bullet is properly expressed in terms of its impulse value (which is equal to the momentum mv of the bullet).

4. The anvil experiment.—*A heavy anvil rests upon a yielding support, and yet it gives a satisfactory base upon which to flatten a piece of iron by a hammer blow.* The enormous force exerted by the hammer lasts for a very short time, say, one ten-thousandth of a second, and, although the anvil is set in motion by this force, the actual distance moved by the anvil in the ten-thousandth part of a second is very small and entirely negligible from the point of view of the blacksmith. The anvil continues to move, however, long after the hammer blow, and as it continues to move it compresses its elastic support for, say, a tenth of a second. But to stop the anvil in a tenth of a second the average force exerted on the anvil by the supporting structure need be only one thousandth as great as the average value of the force (exerted by the hammer) which set the anvil in motion in one ten-thousandth of a second.

5. The coin and card experiment.—Place a small card horizontally on the end of the finger and on top of the card place a small coin. A quick thump against the edge of the card drives it out from under the coin, and the coin is left on the end of the finger. During the very short time required for the card to slide out from under the coin, the card exerts a forward drag on the coin and imparts to the coin a small velocity; but the coin has time to travel only a very short distance before the card is gone, and the coin, then sliding along on the end of the finger, is very soon brought to rest by friction. The whole distance moved by the coin while being started by the forward drag of the card and while being stopped by the backward drag of the finger is, perhaps, one hundredth of an inch or less.

This coin-and-card experiment is worth while if it is carefully analyzed as above, but, like most paradoxical experiments, it is worse than useless if not accompanied by careful analysis.

6. The cord paradox.—A heavy metal ball B is supported by a small cord c as shown in Fig. 4, and a quick hammer blow, as indicated, breaks the heavy cord C. The hammer causes a

very large tension in cord C for a very short time thus setting the ball B in motion. Then the ball continues to move for a relatively long time, stretching cord c. Thus cord c has a very long time in which to stop the ball whereas cord C had an extremely short time in which to start the ball. Thus if cord C is under excessive tension for a ten-thousandth of a second, and if ball B thus quickly set in motion continues to move downwards for a tenth of a second before it is brought to rest by the increasing tension of cord c, then the average excess tension in cord c (in addition to tension due to the steady pull of the earth on B) will be only one thousandth as large as the average tension in cord C during the ten-thousandth of a second.

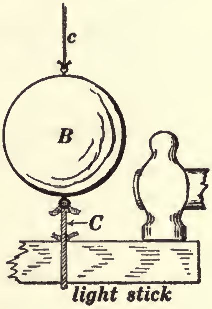

Fig. 4.

UNIFORMLY ACCELERATED TRANSLATORY MOTION.

7. **Experiment with a rolling disk.**—The simplest method of obtaining a close approximation to uniformly accelerated translatory motion with small acceleration is to use a heavy disk with a small axle rolling down an inclined track. According to equation (ii) on page 23, *General Physics*, this disk starting from rest ($v_1 = 0$) should travel a distance proportional to the square of the elapsed time. Lay off from the starting point of the rolling disk a series of distances proportional to 1, 4, 9, 16, 25, etc., and mark the points so located. Then as the disk rolls down the inclined track it will *mark time* as it passes these various points. In order that this experiment may be effective, the distances should be chosen, or the inclination of the track should be adjusted so as to make the rolling disk mark seconds as it passes the various points, and the accuracy with which the disk marks seconds may be appreciated by watching the rolling disk and listening to a telegraph sounder operated by a contact device on a clock pendulum.

8. **Shooting at a falling ball.**—When the initial velocity v_1 of a body is zero, or when it is vertical, we have the ordinary case of a falling body. In this case equation (ii) on page 23, *General Physics*, can be solved by simple algebra, and all calculations can be made by simple arithmetic, the only complication being that v_1 is to be considered negative when it is upwards. When the initial velocity v_1 is not vertical, as in the case of a tossed ball, the falling body is called a *projectile*. In this case the entire argument as given on page 23, *General Physics*, holds good, but geometric addition must be substituted for arithmetical addition in equation (ii). This equation, being interpreted as shown in Fig. 5, means that the position A' of the ball after t seconds is

found by adding together the vectors v_1t and $\frac{1}{2}gt^2$. If the ball at B is released at the instant the projectile leaves the muzzle of the gun at A, then both balls, A' and B', will always be at the same distance $\frac{1}{2}gt^2$ from the line AB. Therefore if the gun is aimed accurately at the point B the two balls will always come together at a point p.

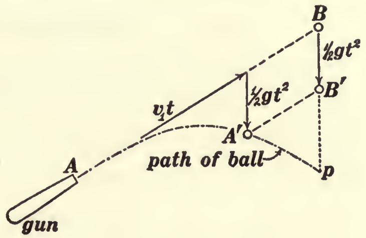

Fig. 5.

The most satisfactory arrangement is as follows: A wooden ball has a hole bored through it so that it will slip over a half-inch brass tube, and a sling with rubber bands is arranged to be released by a trigger and throw the ball along the tube. This gun is sighted at ball B by looking through the brass tube, and as ball A slips off the end of the brass tube it comes against a very light wire bridge which spans across between two brass clips thus breaking an electric circuit. This circuit includes an electromagnet which supports ball B.

CENTER OF MASS.

The study of dynamics always begins with the study of translatory motion. Thus the discussion on pages 3–13, *General Physics*, refers to translatory motion. A *material particle* is an ideal body so small that the only sensible motion of which it is capable is translatory motion, and the term *material particle* is used in dynamics merely to direct one's attention narrowly to translatory motion. Thus, if one is concerned only with translatory motion, one may think of a body of any size and shape as a particle located at the center of mass of the body. *The center of mass of a body is the point at which a single force must be applied* (that is the line of action of the single force must pass through the center of mass) *to produce translatory motion only.*

9. **Experiment with a slim stick.**—The experiment described in Art. 9, page 14, *General Physics*, is quite effective if tried before a class, but every student should try it for himself.

10. **Experiment with hammer and stick** as described on page 15, *General Physics*, is extremely important as elucidating, the definition of center of mass.

11. **Experiment showing the identity of center of mass and center of gravity.**—The center of gravity of a body is the point of application of the single force which is the resultant* of all the forces with which the earth pulls on the various parts of the body. When a body is supported by a single force the line of action of the force must pass through the center of gravity of the body. Therefore the center of gravity of a slim stick can be located by balancing the stick on a knife edge; and the point so found coincides with the center of mass as located in experiment 9.

* See Art. 23 of Franklin and MacNutt's *Elementary Statics*, published by Franklin, MacNutt and Charles, South Bethlehem, Pa.

TRANSLATORY MOTION IN A CIRCLE.

It is important to understand that translatory motion does not mean, necessarily, straight line motion. Thus Fig. 6 indicates a stick performing translatory motion in a circle, and, as explained on page 14, *General Physics*, the entire stick may be thought of as concentrated at its center of mass *C*.

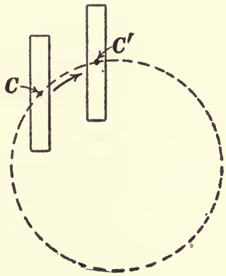

Fig. 6.

When a ball is twirled on a string the ball itself makes one revolution every time it goes round the circle. This is evident when one considers that one particular part of the ball always faces inwards. The motion of the ball is therefore a combination of translatory motion and rotatory motion, but the rotatory motion once established is constant (unchanging in value and about an axis which does not change its direction) whereas the translatory motion is variable (changing in direction).

12. Experiment with ball and string.—The authors have always found it worth while to go through with the discussion of Art. 31, pages 38–40, *General Physics*, in the class room with ball and string in hand. It is well to point out that the pull of the string is the only force acting on the ball, drag of air and pull of gravity being assumed to be negligible.

A ball is tossed through the air; what forces act on the ball after it leaves the hand? If any one in answer to this question should seem to think that the continued motion of the ball indicates a sort of continued driving force exerted by the hand, let him remember that force means always an actual push or pull BY something ON something; and very certainly the hand exerts neither a push nor a pull on the ball after the ball has left the

hand! The gravity pull of the earth continues to act on the ball and the air continues to exert a backward drag on the moving ball; and these continued forces modify the motion of the ball greatly.

But does not something pull outwards on a ball which is being twirled on a string? If such a force exists it must be exerted by something. The pull of gravity is exerted by the earth (although the connecting mechanism between the earth and the ball is invisible), the backward drag of the air on the moving ball is due to actual contact of the air with the ball, and the pull of the string on the ball is evident to anybody. *There is no other force acting on the ball.*

Important example of motion in a circle. The belt wheel.— A wheel is driven by belt as shown in Fig. 7. When the belt is driven at higher and higher speeds, it presses more and more lightly against the face of the wheel and slips more and more easily. At a certain critical speed the belt does not press against the face of the wheel at all.

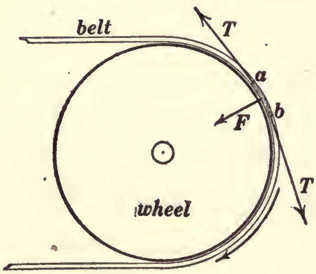

Fig. 7.

Consider a short portion ab of the belt. This portion or particle travels in a circular path as it passes around the pulley. The two forces TT act on the small portion due to the tension T of the belt, the resultant of the two forces TT is a force F acting towards the center of the wheel, and the force F is equal to $\Delta l \cdot T/r$, where Δl is the length of the arc ab and r is the radius of the wheel.*

At zero belt-velocity, the entire force F is effective in pushing the portion ab of the belt against the pulley face. As the velocity of the belt increases, a larger and larger portion of F is used to constrain the portion ab to its circular path or orbit

* See Franklin and MacNutt's *Mechanics and Heat*, page 98.

(that is, to produce the necessary acceleration of ab towards the center of the pulley), and a smaller and smaller portion of F is available for pushing the portion ab against the pulley face.

Let $m \cdot \Delta l$ be the mass of the portion ab of the belt, and let v be the velocity of the belt. Then the inward acceleration of the portion ab is v^2/r and the force required to produce this inward acceleration is $m \cdot \Delta l \times v^2/r$. Therefore when

$$\frac{mv^2 \cdot \Delta l}{r} = \frac{T \cdot \Delta l}{r}$$

or when

$$v^2 = \frac{T}{m}$$

then the portion ab of the belt is not pushed against the pulley at all. It is interesting to note that this critical belt-velocity ($\sqrt{T/m}$) is the velocity at which a wave or bend would travel, with unchanged shape, along the belt. See pages 509–511, *General Physics*.

MOTION OF THE CENTER OF MASS OF A SYSTEM.

The idea of translatory motion can be made to serve as a basis for the description of any complicated motion whatever; all that is necessary is to look upon the moving body or bodies as a collection of particles and consider the varying position, velocity and acceleration of each particle.

A collection of particles treated in this way is called a *system of particles* or simply a *system*. Thus a rotating wheel is a system of particles, and a portion of flowing water is a system of particles.

When the vector sum of all the forces which act on the particles of a system is zero, the center of mass of the system either remains stationary or continues to move with uniform velocity in a straight line. This may be illustrated by considering the rotation of a balanced emery wheel. To say that a rotating wheel is *balanced* means that the center of mass of the wheel lies in the axis of the shaft so that the center of mass remains stationary as the wheel rotates, and no force need be exerted on the shaft by the bearings except the upward force required to support the wheel and axle against the pull of gravity.

When the vector sum of all the forces which act on the particles of the system is not equal to zero, then the center of mass of the system is accelerated in the direction of the resultant of all the forces, the acceleration is proportional to the resultant of all the forces and inversely proportional to the total mass of the system. Indeed in this case we have the equation $F = MA$, where A is the acceleration of the center of mass of the system of particles, F is the vector sum of all the forces which act on the system, and M is the total mass of the system.* This may be illustrated by considering the rotation of an unbalanced emery wheel, an emery wheel of which the center of mass lies at a distance r from the axis of rotation. Then, as the wheel rotates, the

* See Franklin and MacNutt's *Mechanics and Heat*, page 112

center of mass describes a circular path of radius r, the acceleration of the center of mass is equal to v^2/r or to $4\pi^2n^2r$ at each instant, and a side force equal to $4\pi^2n^2rM$ and parallel to r at each instant must act on the axle to constrain the center of mass to its circular path, precisely as if the entire mass of the wheel were concentrated at its center of mass. The force here described is the force which must be exerted on the emery wheel shaft by the bearings, and, of course, the shaft must exert an equal and opposite force on the bearings. This reacting force is usually sufficiently large in the case of a rapidly rotating unbalanced wheel to cause very violent vibrations of the supporting structure.

13. **Emery-wheel experiment.**—Mount an iron disk on a small spindle mounted in bearings and balance the disk with a removable bolt or cap screw near one edge so that the disk can be easily unbalanced by removing this cap screw. Remove the spindle from its bearings and place it upon two parallel level straight edges, and show that it is balanced when the cap screw is in place and unbalanced when the cap screw is removed. Then replace the spindle in the bearings and drive in balanced and unbalanced conditions.

14. **Earth and moon experiment.**—The earth and moon rotate about their common center of mass once every lunar month, and this center of mass describes a smooth elliptical orbit about the sun once a year. This motion of the earth and moon may be illustrated as follows: A large ball and a small ball are tied together with a string, and the center of mass of the two balls (with the string stretched) is marked by a piece of red flannel tied to the string. The balls are then tossed through the air in such a manner as to cause them to rotate and keep the connecting string stretched tight, and *the piece of red flannel describes a smooth curve.* This experiment is not very striking because the eye naturally, and in spite of every effort to the contrary, takes the large ball as a basis of reference, so that the small ball and the red flannel both seem to rotate around the large ball.

SPIN-INERTIA.*

The spin-inertia K of a body about a given axis may be defined as T/α, where T is the torque (unbalanced) which acts on the body and α is the spin-acceleration produced by T (the torque T is of course exerted about the given axis), or K is numerically equal to the torque T required to produce unit spin-acceleration (one radian per second per second).

15. Experiment with a slim stick.—A round stick about two feet long and half an inch in diameter is tapered to a point at one end so that it may be set spinning by thumb and finger. (*a*) Show that a moderately small torque acting for a very short time sets the stick spinning rapidly about its longitudinal axis (OO, Fig. 15, *General Physics*) so that the spin-inertia of the stick about the axis OO is small. (*b*) Show that a much larger torque acting for a longer time produces a much slower spin about a transverse axis (OO, Fig. 16, *General Physics*) so that the spin-inertia of the stick about the transverse axis is large.

16. Direct determination of spin-inertia.—A large wheel is mounted on a shaft of which the radius is r feet, and the shaft is supported in ball bearings. A string is wound around the shaft, a spring scale is attached to the string, a steady pull of F poundals is exerted on the string for, say, exactly 10 seconds $(= t)$, and the length l of string unrolled during this time is measured.

The torque T exerted on the wheel is equal to Fr, where r is the distance from axis of shaft to middle of string, and the spin-acceleration of the wheel is equal to T/K radians per second per second according to equation (7), page 22, *General Physics*. Therefore the spin-velocity s which is gained by the wheel during t seconds is $s = (T/K)t$. Now the average spin-velocity

* Usually called moment of inertia.

during the t seconds is $\frac{1}{2}s$ (see page 23, *General Physics*) and therefore the total number of radians turned during the time t is $\frac{1}{2}st$ or $\frac{1}{2}(T/K)t^2$, or the total number of revolutions turned in the time t is $\frac{1}{2}(T/K)t^2 \div 2\pi$ so that the length, l, of string unrolled during the time t is $l = \{\frac{1}{2}(T/K)t^2 \div 2\pi\} \times 2\pi r$ so that $l = \frac{1}{2}\frac{Tt^2r}{K}$ in which K is the only unknown quantity and its value may therefore be calculated. The result will be expressed in pound (feet).^{2}

Note 1.—The force F may be expressed in "pounds," the torque T in "pound"-feet and the moment of inertia in slug-(feet)^2. See pages 50 and 51 of this volume.

Note 2.—This experiment may be arranged in accordance with the discussion in Art. 34, *General Physics*.

Note 3.—The wheel above mentioned may be a circular disk of solid iron so that its spin-inertia may be calculated by the formula given in the table on page 22, *General Physics* (the spin-inertia of the shaft may be neglected because the above measurement is not very precise) and thus the experimentally determined value of K may be compared with its calculated value.

Note 4.—A very instructive modification of this experiment is as follows: Instead of the wheel let four slim rods be fixed to the shaft like the spokes of a wheel, and determine the value of K of this structure as explained above. Then attach massive steel balls to the spoke-like arms each at a distance ρ feet from the axis of the shaft, and determine anew the spin-inertia K'. Then $K' - K$ will be found equal to $m\rho^2$ where m is the combined mass of the steel balls. It is best, however, according to the authors' experience, relegate quantitative experiments to the laboratory where the student makes all the measurements.

Derivation of the equation $K = \Sigma r^2 \cdot \Delta m$.—The student of mathematics is usually led to interpret the equation

$$\int x^2 \cdot dx = \tfrac{1}{3}x^3 + C$$

thus

$$\left\{\begin{array}{l}\text{The function whose deriva-}\\ \text{tive with respect to } x \text{ is } x^2\end{array}\right\} = \tfrac{1}{3}x^3 + C$$

and from this point of view it is easy to determine the spin-inertia K of a body of regular shape as on pages 594–597, *General Physics*, where the derivative of K with respect to a chosen variable is derived, etc. It is easy to alter this discussion so as to use equation (7), page (22), *General Physics*, namely, $T = K\alpha$, instead of the equation $W = \frac{1}{2}Ks^2$. Consider the added mass $2\pi Ar \cdot \Delta r$; its sidewise velocity (at right angles to r) is rs, its sidewise acceleration is $r\dfrac{ds}{dt}$ or $r\alpha$, and the sidewise force ΔF which must act on it to accelerate it is $\Delta F = 2\pi Ar \cdot \Delta r \times r\alpha$. Therefore a portion ΔT of the total torque $T\ (= K\alpha)$ which is acting on the body is used to accelerate the added material, namely,

$$\Delta T = r \cdot \Delta F = 2\pi Ar \cdot \Delta r \times r\alpha \times r.$$

Therefore since $\Delta K = \Delta T/\alpha$ we have $\Delta K = 2\pi Ar^3 \cdot \Delta r$ so that the derivative of K with respect to r is $2\pi Ar^3$.

The more general equation $K = \Sigma r^2 \cdot \Delta m$ may be derived as follows: Let us consider a small particle of the body of mass Δm at a distance r from the axis of rotation. The sidewise velocity of Δm (at right angles to r) is rs and the sidewise acceleration of Δm is $r\alpha$. Therefore the sidewise force which must act on Δm is $\Delta F = \Delta m \times r\alpha$ and the torque value of this force about the axis of rotation is $\Delta T = \Delta m \times r\alpha \times r$, so that the total torque T acting to produce spin-acceleration is $T = \alpha\Sigma r^2 \cdot \Delta m$ so that $K = \Sigma r^2 \cdot \Delta m$.

THE GYROSCOPE OR GYROSTAT.

It is scarcely possible to bring out the simple meaning of the discussion of Figs. 39*a*, 39*b* and 39*c*, *General Physics* without showing the experiment as follows:

17. Experiment with a bicycle wheel.—(*a*) Hold a bicycle wheel (*not spinning*) as indicated in Fig. 8, supporting both ends

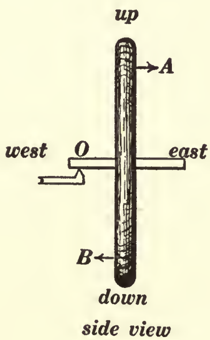

Fig. 8.

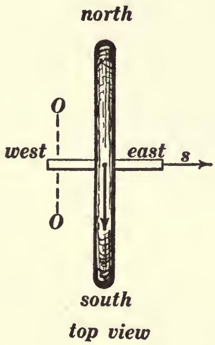

Fig. 9.

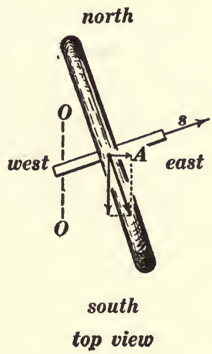

Fig. 10.

of the spindle by the hands. Release the spindle at one end and keep it supported at end O, catch it again immediately and call attention to the fact that *eastward velocity has been given to every particle in the upper half of the wheel and westward velocity to every particle in the lower half of the wheel by the unbalanced torque due to the gravity pull of the earth on the wheel.*

(*b*) Hold the *spinning* wheel and release it in the same way, catch it again immediately and call attention to the fact that *eastward velocity has been given to every particle in the upper half of the wheel and westward velocity to every particle in the lower half of the wheel by the unbalanced torque due to the gravity pull of the earth on the wheel.*

(*c*) Hold the *spinning* wheel and release it as before and allow

it to precess steadily, and point out the meaning of Fig. 38, *General Physics.*

The essential steps in the analysis of the sidewise accelerations of the particles of a rotating disk in precession are given on pages 168–169 of Franklin and MacNutt's *Mechanics and Heat.*

There are two distinct aspects of the problem of the simple gyroscope, namely, (*a*) To determine the torque required to produce a specified precession, and (*b*) To determine the precession produced by a given torque. The first aspect of the problem is very much simpler than the second. Indeed the first aspect is a *problem in differentiation*, that is to say, it turns out to be such when it is fully developed; whereas the second aspect turns out to be a *problem in integration.* Now any problem in integration involves the evaluation of integration constants (on the basis of initial conditions supposed to be given), and the oscillatory movements of the gyroscope as very briefly described on pages 42 and 43 of this volume are related to these integration constants.

Why is it that one does not use the moment of inertia about the axis T in Fig. 37, *General Physics*, in the equation $T = K\alpha$? One certainly would use the moment of inertia about T if the wheel were not already spinning! Answer: One does use the moment of inertia about T and several things not mentioned in the bob-tailed theory (the simple theory as given in *General Physics*), *unless one thinks of problem* (*b*) *as the simple inverse of problem* (*a*).

WHAT IS LACKING IN THE DISCUSSION OF FIGS. 14*a* AND 14*b* IN *General Physics*.*

It is assumed in the discussion of Figs. 14*a* and 14*b* and again in the discussion of Figs. 37 and 38 in *General Physics* that spin-velocities can be added by the parallelogram law. The proof of this is outlined in Franklin and MacNutt's *Mechanics and Heat*, page 176.

It is the validity of the parallelogram law that justifies the representation of spin-velocity by a line or arrow as specified on page 19, *General Physics*. Similarly the parallelogram law must be valid for the addition of torques to justify the representation of a torque by a line or arrow as specified on page 19 of *General Physics*. The demonstration of the parallelogram law for the addition of torques is given in Franklin and MacNutt's *Mechanics and Heat*, page 177.

It is utterly inadequate to say that spin-velocity and torque are "directed quantities and therefore to be added by the parallelogram law."† This is evident when we consider that a displacement of a body through a certain angle ϕ about a given axis is not to be distinguished in its mode of specification from such quantities as spin-velocity and torque, whereas angular displacements cannot be added by the parallelogram law.

* One thing that is lacking grows out of the assumption that problem (*b*) as described on page 32 is the simple inverse of problem (*a*). This deficiency most beginners will accept uncomplainingly, we are sure of that because we have mentioned integration in connection therewith. We refer here, in particular, to deficiencies that are comparatively easy to supply, but which simply must not be lost sight of by accepting plausibility for rigor.

† Professor Rettger's review of the second edition of Dadourian's *Analaytical Mechanics* (*Science*, August 26, 1916, page 279) calls attention to a fault which becomes more serious in Professor Dadourian's later attempt at defense. Plausibility is in many cases necessary in the presentation of a mathematical subject to young men, but to think of plausibility as the same thing as rigor is fatal.

One of the best discussions of vector quantity is to be found in the first chapter of Abraham and Föppl's *Theorie der Electrizität*, Vol. I, third edition, Leipzig, 1907.

Science, even in its elements, presents serious difficulties. Anyone who thinks otherwise knows only pseudo-science as P. G. Tait has said,* and pseudo-science is the bane of science teaching.† Indeed the most serious fault with our physics teaching is that our physicists who know all that has been thought and all that has been done in physics (and there are many of our physicists who do) hold themselves aloof while half-educated men, as in the recent rehash of elementary mechanics in *Science*, talking as oracles from out of their dignified institutional settings, make pi of everything!

* See introduction to Tait's *Heat*.

† The most convincing statement of this fact has been made by a student of the Classics. See remarkable article by Paul Shorey in the *Atlantic Monthly* for June and July, 1917.

THE USE OF THE GYROSCOPE AS A RUDDER CONTROL.

Consider a freely precessing gyroscope, the precession being due to the pull of gravity. If the precession is hindered by letting the end of the gyroscope axle (see Fig. 39*c*, *General Physics*) come against a stop, the gyroscope drops at once like any ordinary inert body. In order that the gyroscope may stand out horizontally in spite of the downward pull of gravity, the gyroscope must be free to precess about a vertical axis.

An important use of the gyroscope is for automatically controlling the rudder of a torpedo or of a flying machine. In such a case the gyroscope constitutes a frame-work which keeps an unchanged direction in space, and a sudden veering of the torpedo or flying machine to right or left brings the gyroscope frame against a lever which opens a valve and admits compressed air to a piston which shifts a rudder. To exert a force in opening the valve the gyroscope must be entirely free to precess in a plane at right angles to the direction of the force, exactly as in the case of a gyroscope precessing under the pull of gravity as above explained.

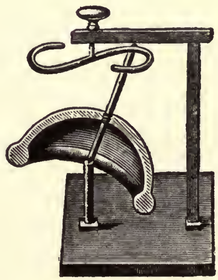

Fig. 11.

Experiment 18.—Support one end of the axle of a spinning bicycle wheel in the hand, allow the other end of the axle to come against the side of a vertical post, and call attention to the fact that the axle drops exactly as if the wheel were not spinning, or would so drop, if the powerful reaction did not move the hand-support sidewise and if it were not for the great friction due to the very large force with which the moving end of the axle is pressed against the post.

19. Curious gyroscope toy.—When the moving end of the axle of a precessing gyroscope comes against an obstacle (even

when no *outside* torque is acting to push the axle against the obstacle) the axle, if it is rotating with the wheel, starts to roll along the obstacle. This motion (precession) of the axle causes a reaction which pushes the axle against the obstacle so that the rolling axle will follow an obstacle of almost any shape. Fig. 11 shows a very interesting toy in which the axle of a spinning top rolls round and round the entire boundary of a double spiral made of a strip of metal.

WHAT MAKES A HARD-BOILED EGG RISE ON END WHEN SPUN ON ITS SIDE?

Experiment 20.—Spin a sharp-pointed top with its axle inclined and note its motion of precession. This precession is caused by the tipping action or torque action produced by the pull of gravity, and the precession produces a *reaction* which is just sufficient to keep the axis of the top at a constant inclination.

Increase the speed of the precessional motion by holding a thin rod above the top and helping the precession. This increased precession produces *more than enough reaction* to support the inclined top against gravity, and therefore the axis of the top rises to the vertical.

The rolling of the blunt end of a spinning top (with its axis inclined) on the table hastens the precessional motion of the top by producing a side force against the lower end of the axle of spin, and this hastening of the precession soon brings the axis of the top into a vertical position.

The above effects can be easily shown by spinning two tops which are alike except that one has a very blunt point and the other a sharp point. A hard-boiled egg spun on its side quickly stands up on one end, the action being the same as in case of the blunt pointed top.

THE SCOPE OF DYNAMICS.

It may seem that an excessive amount of space is devoted to gyroscopic motion in *General Physics* and in this *Calendar*, but rational mechanics is an important subject and it must be adequately treated! The plan which is commonly followed in our technical schools is to give a course in analytical mechanics to supplement an altogether inadequate course in elementary mechanics; and this plan is worse than useless for two reasons, namely, (*a*) Because analytical mechanics, the oldest branch of theoretical physics, has less to do with engineering and less to do with research than any other branch of mathematical physics, and (*b*) Because the limitations of the engineering student and the limitations of the engineering teacher have together led to the development of a course in analytical mechanics which it is a kindness to call mere formal nonsense. It sickens the brilliant student and stupefies those who are not brilliant; and a close competitor is the accepted course in thermodynamics which has been developed under the same hopeless limitations; but, according to our experience, there is no group of teachers so stiffly proud of what they do as the teachers of theoretical mechanics and thermodynamics in our technical schools.

What are authors to do who arrange an elementary treatise on dynamics? Translatory motion must be taken up first, then rotatory motion, then oscillatory and wave motion, and then, were it not for the limitations of the student, a touch of statistical mechanics which sticks its nose into Everything! But we know the imperturbable satisfaction of Deans of Engineering; what are we to do? Having come to detest the technical school—no, not the school but the elaborate parts of its curriculum, we cannot be satisfied merely to expound the elementary theory of dynamics (we are willing to stop short of statistical mechanics) and contemplate mortality tables!

The old conflict in education was between the older classics and science, but the conflict is now between science and pseudo-science. We have the watery pseudo-science of the high school and college which is intended to be enticing and plausible; and we have the pseudo-science of the technical school which is exacting but unintelligible, Dryer than Dust. It would desiccate the Ocean if it were not far Up in the Air.

THE BRENNAN MONO-RAIL CAR.

A gyrostat wheel is mounted on a horizontal spindle, this spindle is carried in a frame *FF*, Fig. 12, this frame is free to turn about a vertical axis, being pivoted in an outer frame *GG* which is fixed to a platform *PP*, and this platform stands on two pointed legs *O* (one leg is behind the other in the figure).

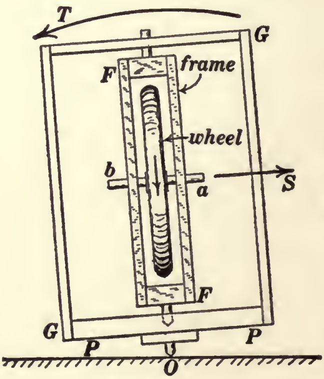

Fig. 12.

The platform *PP* with attached frame *GG* is shown as slightly inclined to the left in Fig. 12 so that the pull of the earth on the entire arrangement exerts upon it a torque *T*, and the effect of this torque is to make the spinning wheel precess, bringing the end *a* of the spindle towards the reader. The platform does not fall over for a comparatively long time (while the precession is taking place as stated), and the essential feature of the Brennan balancer is a device for automatically *hastening* this precession. The reaction of the unhastened precession is just sufficient to balance the torque *T*, but the reaction of the hastened precession is more than enough to balance *T*, and therefore the hastened precession lifts the end *b* of the spindle. Indeed in the Brennan balancer the lifting of the end *b* of the spindle is always enough to reverse the tilt of the platform and reverse the torque *T*, whereupon the precession is reversed. This reversed precession is allowed to continue until the end *a* of the spindle has come back into the plane of the paper, and then the reversed precession is hastened, the platform is tilted

back to the left (as shown in the figure), the end a of the spindle comes forwards to the plane of the paper, the precession is then again hastened, and so on over and over again. A number of devices have been used for automatically hastening the precession as stated.

The only advantage of the mono-rail is that a car (if it is kept erect) would *run with less vibration on one crooked rail than on two crooked rails!* It certainly is better, however, to build two *straight rails* and use ordinary cars than it would be to save a little on a crooked mono-rail and spend a lot on complicated and more or less unreliable balancing mechanisms! In fact the great problem in heavy-traffic railroading is the problem of making the track strong enough to carry the enormously heavy engines and trains, and mono-rail construction would be an engineering absurdity.

The gyro-platform.—Two Brennan balancers can be arranged to keep a platform very nearly level on board ship, and such a gyro-platform may come to have important uses.

Experiment 21.—A person sitting on the platform PP in Fig. 12 can maintain his balance easily by properly hastening the precession of the heavy lead-rimmed bicycle wheel which has been set spinning by hand, especially if the two platform legs are short.

22. The Schlick device for the steadying of a ship at sea.—Figure 13 shows a demonstration model of this device. The ring swings freely about a horizontal precession axis pq, and the axis of spin of the wheel is at right angles to pq as shown. The boat is represented by the flat board AB which stands on two legs OO.

Take the strings ss in the hands and exert a repeatedly reversed torque on the model, tending to rock the boat back and forth about OO as an axis.

Let us suppose that the axis of spin of the wheel is vertical

at the beginning, then pulling on either string does not* tip the boat but causes precession about the axis pq. A steady pull on either string soon brings the axis of spin into a horizontal fore-and-aft position, and then the boat tips freely. The gryostat is only effective in preventing the tipping or rocking of

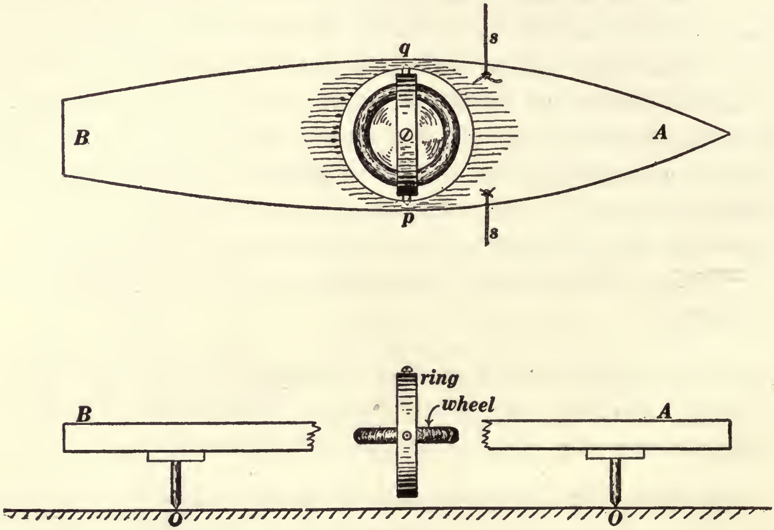

Fig. 13.

the boat when repeatedly reversed pulls are exerted on the strings, and the intensity and duration of a single pull must not be sufficient to bring the axis of spin into the horizontal fore-and-aft position.

23. **Gyrostatic elasticity.**—A spinning gyroscope is held in the position shown in Fig. 14, and we will speak of the end of the axle which is opposite to O as the outer end. The support of the outer end is suddenly removed. To establish the precessional motion Ω the ring and wheel must of course be set in motion about a vertical axis, and this means inertia reaction whose effect, while it lasts, is exactly equivalent to an outside torque opposing

* Except for an elastic-like yielding which immediately recovers when the pull ceases. See following discussion of gyrostatic elasticity.

Ω. Therefore the outer end of axle and ring drops slightly while precessional motion is being established. By the time Ω has reached its normal mean value (the value due to gravity torque on ring and wheel) the downward velocity of outer end is a maximum and the downward momentum of ring and wheel carries the outer end still farther downwards and doubles the value of Ω. Then the unbalanced reaction of this doubled Ω (the reaction due to mean value of Ω is just sufficient to balance the gravity torque) lifts the outer end to its initial level and reduces Ω to zero as at the beginning (the outer end having moved forwards, however, in the direction of Ω). The action as described is repeated over and over again.

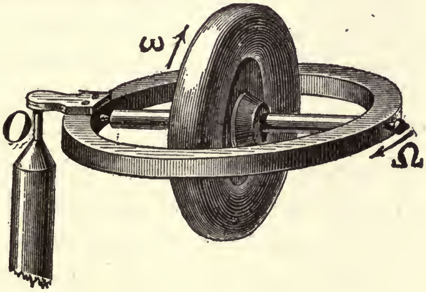

Fig. 14.

When one of the strings in Fig. 13 is pulled suddenly the boat tips slightly while the precession is being established, and if the pull then ceases the momentum which is associated with the established precession causes the precession to persist for a short time, and the unbalanced reaction of this continued precession brings the boat again to a level position. If the strings are removed in Fig. 13 a sudden blow causes the boat to oscillate back and forth on its two legs exactly as if the boat were held in a vertical position by an elastic spring, and this oscillatory motion of the boat is accompanied by an oscillatory precession of the gyrostat.*

* A very simple theoretical discussion of this kind of oscillatory motion is given by W. S. Franklin, *Physical Review*, Vol. XXXIV, pages 48–52, January, 1912.

CONSTANCY OF SPIN-MOMENTUM OF A CLOSED SYSTEM.

When a body or system of bodies is not acted upon by any forces from outside the body or system, the body or system of bodies constitutes what is called a *closed system.* The only force actions in a closed system are the mutual force actions between the parts of the system.

The total momentum of a closed system cannot change. This is evident when we consider two things, namely, (*a*) That an unbalanced force acting on a body causes the momentum mv of the body to change, and that the rate of change of the momentum $\frac{d(mv)}{dt}$ is equal to the force $m\frac{dv}{dt}$ because m is a constant, and (*b*) That any mutual force action between two bodies in a system consists of a pair of equal and opposite forces so that the rates of change of the momenta of the two bodies must be equal and opposite so that their combined momentum does not change.

The spin-momentum of a body is equal to Ks, where K is the spin-inertia of the body and s is its spin-velocity. In a closed system a mutual torque action between two parts of the system means the exertion of equal and opposite torques on the two parts, and equal and opposite rates of change of spin-momentum by the two parts. Therefore the total spin-momentum of a closed system cannot change.

24. Experiment with a pivoted stool.—A platform or stool is supported on a ball-bearing pivot. Stand on this platform with weights in the hands and with arms outstretched, and have an assistant set you rotating slowly about a vertical axis. Then draw in your arms thus greatly decreasing the spin-inertia K of the system. The spin-momentum Ks remains constant and the spin-velocity s is greatly increased when K is reduced.

25. Experiment with emptying bowl.—A deep bowl, preferably of glass, is filled with water and the water is set slowly rotating by stirring the water with a stick. A hole at the center of the bottom of the bowl is then opened and the water is allowed to run out. The water in the bowl flows to some extent towards the axis of the bowl and the spin-inertia of the system (water in the bowl and the long "rod" of water which is to be thought of as coming out of the hole) decreases greatly. Therefore the slow rotatory motion of the water is greatly increased as shown by the production of a deep whirl at the center of the bowl. See description of cyclone and tornado, page 91, *General Physics*.

26. Experiment with pivoted stool and heavy rimmed bicycle wheel.—Stand on the pivoted stool, hold the bicycle with its axis horizontal and set it spinning rapidly. Then when the axis of the spinning wheel is brought into a vertical position your body will be set rotating in a direction opposite to the spinning wheel. Assuming the pivot of stool to be frictionless no torque about a vertical axis can be exerted on your body from outside, and therefore the spin-momentum of the pivoted system about a vertical axis must remain what it was at first, namely, zero.

STATIC AND DYNAMIC BALANCING.

A wheel and axle is said to be *statically balanced* when the center of mass (or center of gravity) of the whole lies on the axis of the axle. Such a wheel and axle will stand indifferently in any position when the axle rests on two parallel, straight, horizontal rails.

A wheel and axle is said to be *dynamically balanced* when the axle has no tendency to quiver when the whole is spinning rapidly.

Figure 15 shows a body B mounted on a shaft, and the center of mass C of the whole is on the axis of the shaft; that is to say, the body is statically balanced. But if the shaft

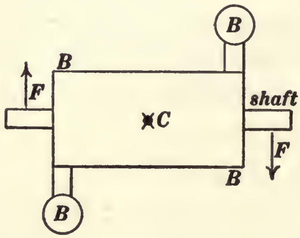

Fig. 15.

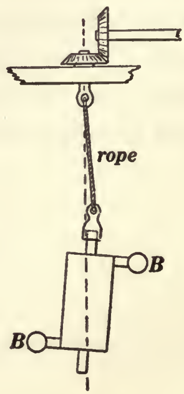

Fig. 16.

is supported in bearings and the body set rotating, the bearings must exert very considerable forces upon the shaft, and the arrows FF show the directions of these forces for the given instantaneous position of the body; or if hung by a rope as shown in Fig. 16 and set spinning, the shaft will take the position shown.

Dynamic balancing of a dynamo armature.— A thin disk or narrow wheel and axle can be brought into dynamic balance by placing the axle on horizontal rails and adding material to or taking material from one edge or the other until the wheel and axle is in static balance. But a long body like a dynamo armature, having been statically balanced, must be hung as shown in Fig. 17 and set spinning rapidly about a vertical axis. If the armature is dynamically out of balance opposite sides *a* and *b* of the shaft will be whitened by chalk held lightly against the spinning shaft. Equal weights must then be added at *c* and *d* (or taken away from *e* and *f*); and this operation is repeated until the balancing is satisfactory.

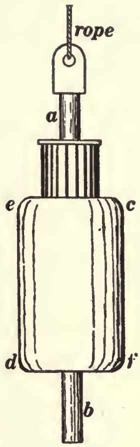

Fig. 17.

ONE, OILER, KNEW IT ALL!

The motion of a symmetrical spinning body like a wheel or a top is extremely simple as compared with the motion of a non-symmetrical body, and any one who cannot digest Euler's (pronounced Oiler's) equations should go to Poinsot's wonderful paper entitled *Theorie Nouvelle de la Rotation des Corps*, or make for himself one of the non-symmetrical tops which Maxwell devised many years ago. It is a common remark among thorough students of rigid dynamics that Euler knew it all! and he did, in fact, know it all.

Anyone can know what Euler knew by taking some pains, and to one who knows what Euler knew it is extremely funny, and more than funny, to read the following letter which was addressed to the editor of an important scientific journal by a director of the Aëro Club of America asking an utterly meaningless question about the simplest aspect of rotatory motion with a pretense of forty thousand dollars' worth of seriousness; for, in fact, the Cash Prize faded to nothing when investigation was made.

Dear Sir: I respectfully state that the following question is before the members of the Aëro Club of America for solution and I would appreciate it very much if you would kindly put the question to the readers of ——— through its valuable columns:—

"What is the acceleration of precession when mass spins and precesses with the same radius vector, and in the same plane, tangential to the earth's surface?"

The above question is important and is put in consideration of $40,000 Cash Prize offered.

I remain, Yours for the advancement of science,

——— ———, Director.

Ours for the advancement of science! Yes, now, ours. He means it with all the good will he can muster without cost or pains. But he was at one time yours, gentle teacher, whether of the technical school or college, and he left you, untouched by those kindly but rigorous exactions, which when shot through and through with sympathetic intelligence *do* bring young men to the pains of those who really learn and brace them for the grief of those who are wise.

WORK AND POWER.

The difficulty of the precise ideas of work and power as applying to actual conditions and things is amusingly illustrated by the following problem which we gave to a group of engineering students at the end of a half-year's course in elementary mechanics. "A cart moves due northwards at a velocity of $5\frac{1}{2}$ feet per second. A man pushes vertically downwards on the cart with a force of 200 pounds, and a mule pulls due northwards on the cart with a force of 50 pounds. Find the rate at which the man does work and the rate at which the mule does work." We deliberately refrained from naming the part of the man's body which exerted the force on the cart! In answer to the question 44 per cent of the young men found that the man developed 2 horse-power and the mule developed $\frac{1}{2}$ horse-power. Truly, mule driving would be strenuous labor for our pampered college students!

Another example illustrates the dislike of many young men for the hardest labor in the world, namely, thinking! "The force required to tow a boat is, let us say, proportional to the velocity of the boat. To tow the boat at a velocity of 2 miles per hour requires 2 horse-power; how much power would be required to tow the boat at a velocity of 4 miles per hour?" More than 95 per cent of the class accepted unthinkingly the suggestion of simple proportion and got 4 horse-power as the answer!

Kinetic energy and the engineer's system of units. We usually speak of the system of units which is based on the foot as the unit of length, the earth-pull-on-a-one-pound-body-in-London as the unit of force and the second as the unit of time as the *foot-slug-second system* (f.s.s. system) for the sake of the sharp differentiation which is thereby made between this system

and the foot-pound-second system (f.p.s. system). *The units of any system may be used in any equation in mechanics (including hydrostatics and hydraulics) if the equation is in its simple generalized form.* Thus in the f.s.s system the kinetic energy of a moving body in foot-"pounds" is equal to $\frac{1}{2}mv^2$ where m is the mass of the body in slugs and v is its velocity in feet per second; the pressure p due to a column of fluid is $p = hdg$, where, in the f.s.s system, p is expressed in "pounds" per square foot, h is the height of the fluid column in feet, d is the density of the fluid in slugs per cubic foot and g is the local acceleration of gravity in feet per second per second. Thus if the density of water is precisely 62½ pounds per cubic foot, it is exactly $\frac{62.5}{32.174}$ slugs per cubic foot, and the equation $p = hdg$ gives the pressure exactly in London "pounds" per square foot if g is the local value of gravity. The numerical precision which is required for most practical purposes (research or engineering) does not justify us in carefully distinguishing between the acceleration of gravity in London, 32.1740 feet per second per second, and the acceleration of gravity at any other place on the earth—but precision of thought demands that we make this distinction. See page 4 of this volume for further discussion of this subject.

The principle of the conservation of energy.—Standing before the class take a stone and lower it from a high position A to a low position B, thus getting work or energy out of the stone; then bring the stone back from B to A, slipping it behind the back in the hope or pretense of getting it back to A with a small expenditure of work! This procedure will go far to give to the student a clear idea of the principle of the conservation of energy as discussed in Art. 48, pages 68–70, *General Physics*, and it strongly suggests the point of view of every perpetual motion promoter, namely, a vague and wholly unintelligent expectation of success or downright cheating.

HYDROSTATICS.

27. Experimental demonstration of Pascal's principle.—Use the arrangement described in Art. 51, pages 73–74, *General Physics*.

Pascal's principle and the idea of hydrostatic pressure. The force ΔF which is exerted by a fluid at rest on an exposed plane area Δa is at right angles to Δa, *the value of* ΔF *is the same whatever the direction of* Δa (Pascal's principle), and the ratio $\frac{\Delta F}{\Delta a}$ approaches a definite limiting value as Δa and ΔF approach zero (ΔF is more and more nearly in exact proportion to Δa when Δa and ΔF grow smaller and smaller).

It is difficult and in fact unnecessary to distinguish sharply, in this outline, between the elements which are based upon or suggested by observation and experiment and the elements which are of a purely postulate character, but the outline as it stands *must* be considered and fully grasped by one who wishes to have a clear idea of the hydrostatic pressure at a point in a fluid.

Having established the idea of hydrostatic pressure at a point in a fluid, it is easy to show that the pressure must have the same value everywhere in a body of fluid at rest when the fluid is not acted upon by an outside force like gravity. Consider any portion of the fluid in the form of an elongated rectangular parallelopiped or prism. The forces exerted by the surrounding fluid against the sides of the prism balance each other and these forces have no components parallel to the axis of the prism. Therefore the forces exerted by the surrounding fluid on the ends of the prism are equal and opposite, and consequently the hydrostatic pressure has the same value at the ends of the prism.

Compare with the above the following line of development, which is the best we know of in any elementary treatise on physics:

(*a*) It is pointed out that ΔF is at right angles to Δa.

(*b*) Pressure is then defined as force per unit area.

(*c*) Then hydrostatic pressure is characterized as being the same in every direction. (If the author had stated that the value of ΔF is the same whatever the direction of Δa the statement would be intelligible, but it is non-sense* to say that "the pressure is the same in every direction.")

(*d*) Then, after discussing the non-uniform distribution of pressure in a fluid under the action of gravity, the author states Pascal's principle as follows: "Pressure is transmitted equally in all directions, or, if the pressure at any point is increased it is increased everywhere throughout the fluid mass by the same amount." The first clause is vague and unintelligible, and the second clause is true only when properly qualified as to the time required for the re-distribution of the pressure and as to incompressibility of the fluid when an outside force like gravity acts on the fluid. But a statement which has to be qualified cannot be a statement of a *principle*. For example, a well known writer on elementary mechanics illustrates equality of action and reaction by pointing out that the pull of a mule on a rope and the backward pull of a canal boat on the rope are equal *if the weight of the rope is negligible*, from which it would be proper to conclude that Newton's third law of motion is not true because ropes always do have weight!

The only comment needed on the above statement of Pascal's principle (?) is to quote Pascal himself, and curiously enough the passage which best serves our present purpose (which is to show the absolute necessity of mathematical thinking *in the mathematical sciences*) is used by Sir William Hamilton in his "famous and terrific diatribe against mathematics." † Speaking

* Let no one who is not familiar with stress as a six-fold complex presume to question this statement.

† This phrase is borrowed from a recent adoration of mathematics by an "institutionally" well known mathematician. It is, of course, ridiculous. See page 34 of this volume for further comment, and consider carefully what is meant by an "institutional" reputation as applying to an individual. As applying to the present case we would change the word *pi* on page 34 to *mush.*

of the recognition of principles Pascal says that "Nothing is wanted beyond a good sight; but good it must be, for principles are so minute."

28. Pressure in a liquid depends only on depth.—A valve V, Fig. 18, is held up by a string which is attached to one end of the beam of a balance scale, and the weights on the scale pan are adjusted so that the valve opens when the wide vessel W is filled with water to a certain level ll. The valve is then found to open when the narrow vessel N is filled to the same level.

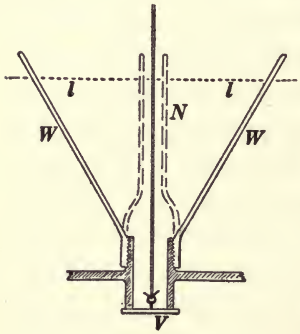

Fig. 18.

The use of engineer's units in hydrostatics. The use of the two equations (26) and (27) on page 75, *General Physics*, is very confusing. It is much better to "go the limit" and use f.s.s units for all of the quantities in equation (26), expressing d in slugs per cubic foot. Then equation (27), which is strictly true in London only, may be discarded. See discussion of equation (26) on page 51 of this volume.

Boyle's law.—Apparatus for the verification of Boyle's law is familiarly known, but according to the authors' experience this experiment should be done by the student in the laboratory.

29. Experiment on compressibility.—Plug the outlet of an ordinary bicycle pump. Stike the pump handle with a small mallet (a) with the pump barrel filled with air and (b) with the pump barrel filled with water. This experiment shows very strikingly the enormous difference between the compressibilities of air and water, indeed in this experiment the water seems to be totally incompressible and the student can understand why, for most practical purposes, water is thought of as incompressible.

Instead of a bicycle pump with its slim piston rod, a one-

inch steel plunger fitting accurately in a one-inch barrel may be used. In this case one may strike the plunger a heavy blow without damaging the apparatus.

30. Torricelli's barometer.—Fill a clean glass tube (closed at one end) with clean mercury and invert in a tumbler of mercury. Show that difference of level of mercury in tumbler and tube is the same whether the tube be vertical or inclined—if the space above the mercury contains but little air.

The student should have an opportunity to take hold of the barometer tube and lift it off the bottom of the tumbler to feel the unbalanced downward pressure of the air on the top of the tube.

31. Archimedes' principle.—A solid cylinder C fits accurately into the cylindrical pail P, the two are hung from one pan of a balance, and the weights are adjusted to give equilibrium. The cylinder C is then submerged in water, and equilibrium is again established by filling the pail P with water. The buoyant force of the water on C is equal to the weight of the water in the pail.

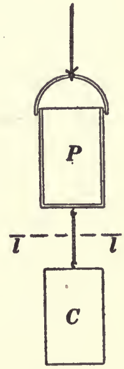

Fig. 19.

32. Capillary elevation and depression.—The capillary rise of water and the capillary depression of mercury in small glass

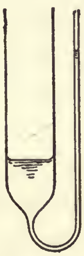
Fig. 20.
Containing water.


Fig. 21.
Containing mercury.

tubes may be shown by placing in the lantern (horizontal projection arrangement) glass tubes arranged as shown in Figs. 20 and 21.

33. Surface tension experiments.—(*a*) Mix alcohol and water to get a liquid of the same density as olive oil, place the alcohol-water in a lantern cell and drop in it several drops of olive oil. Some prefer to use drops of carbon bisulphide floating in a carefully adjusted salt solution.

(*b*) Place a clean horizontal glass plate in the lantern (vertical projection arrangement). On the plate place colored water to a depth of about a millimeter. Allow a drop of alcohol to fall on the middle of the plate. The water draws itself away from the spot where the alcohol falls leaving this spot nearly dry because the surface tension of the pure water is greater than the surface tension of the mixture of alcohol and water.

(*c*) A fine German silver wire lies flat against a glass plate which is placed horizontally in a lantern (vertical projection arrangement). A thin layer of machine oil is poured on the plate and the oil is seen to heap itself up near the wire because of capillary action. Heat the wire by an electric current. The oil draws itself away from the wire leaving the surface of the glass nearly free from oil near the wire, because the surrounding cool oil has a greater surface tension than the hot oil near the wire.

(*d*) Place a shallow basin with glass bottom, thoroughly cleaned and rinsed, in the lantern (vertical projection arrangement). Fill with clean water. Skim the surface of the water by blowing a portion of it out of the basin. Scrape a few very small shavings of camphor gum and allow them to fall into the basin. See *General Physics*, page 83.

Place in the basin above mentioned a very small cork boat with a split stern into which split is inserted a bit of camphor gum. See *General Physics*, page 83.

Note.—The above mentioned glass basin should be made by cementing *two* glass rings to a glass plate. Cut one of the rings from a bottle 4 or 5 inches in diameter and the other from a bottle 5 or 6 inches in diameter. Grind one edge of each ring flat and cement to plate using a minute quantity of gutta percha

or rubber cement or marine glue. Then in the skimming process above mentioned the water that is blown off collects in the annular space. Both experiments as above mentioned (and especially the experiment with the cork boat) show more satisfactorily if the basin is heaping full of water so that the water surface is convex near the walls of the basin.

34. **Experiments on oil flotation.**—Everyone is familiar with the fact that water does not spread over an oily surface. Thus drops of water stand without spreading on an oily surface of glass whereas they spread out and cover a very clean glass surface. Exactly these same effects are shown by a glass surface under water. The water spreads under a bubble of air and detaches it from a clean glass surface whereas the water does not spread under a bubble of air on an oily glass surface, or, in other words, the bubble of air clings to the oily surface. This may be easily shown as follows: (*a*) Agitate the water in a clean glass tumbler, using a clean egg beater, and the air bubbles all rise to the surface and leave the walls of the tumbler clear, (*b*) Do the same in a tumbler with greasy walls and numerous air bubbles are left clinging to the tumbler walls.

There seems to be a marked difference in the tendency of oil to cling to small particles of sand and to small particles of iron pyrite or zinc blende. Place a mixture of fine sand and powdered iron pyrite into a tumbler of water, add a drop or two of oil and agitate with an egg beater as above. The particles of pyrite become oily, bubbles of air cling to them and they float, whereas bubbles of air do not cling to the particles of sand and they sink. This experiment illustrates the oil flotation process which is now extensively used for separating particles of ore from particles of sand and rock.

HYDRAULICS.

The limitations of mechanics.—It is important to point out to the student the fact that the science of hydraulics deals almost entirely with ideal types of fluid motion, as explained on pages 85–87, *General Physics*, because, in the study of any branch of science, nothing is more important than to understand to some extent the limitations of the method used.

The actual phenomena of fluid motion are erratic in character, visibly erratic, and it is quite certain that these phenomena can never be correlated by the classical method of mechanics.* This statement, it must be admitted, represents a very recent point of view, one that is accepted only by a small group of physicists, and we may be permitted therefore to set it forth more strikingly by quoting from a well-known engineering writer who is still deeply imbued with the ideals of the classical method of mechanics. The laws of fluid motion, he says, have not yet been discovered, although the movements of the heavenly bodies have been completely formulated. This statement shows that the writer assumes the existence and expects the discovery of rigorous correlations in the *minutiæ* of fluid motion, only, he would probably deny that *laws of fluid motion* would have anything to do with such ephemeral and insignificant phenomena. Every phenomenon that we contemplate in this world of ours, even such extremely regular phenomena as the motion of the planets, if examined with extreme care shows a substratum of erratic behavior. In the case of fluid motion, however, this erratic action rises to a level where it enters into human values (if we may be permitted to use that word to designate the things we consider and must make allowance for in our daily life) as exemplified most strikingly in the phenomena of meteorology.

* Statistical mechanics is indeed not a branch of mechanics proper. It is a *theoretical structure* with a *purely postulate basis*, and in its bearing on upon laboratory or research work it is a part, or, indeed the whole of the *atomic theory*.

Read in this connection:

(*a*) Pages 119–123, *General Physics*, where the method of thermodynamics is contrasted with the method of atomics (the atomic theory),

(*b*) Pages 322–325, *General Physics*, where the method of atomics is contrasted with the method of mechanics, and

(*c*) A brief and very simple paper on Statistical Physics, in *Science*, Vol. XLIV, pages 158–162, August 4, 1916.

35. Turbulent fluid motion. The sensitive flame.—Light an ordinary fish-tail gas jet and increase the gas pressure (the velocity of the gas in the jet) more and more until the flame becomes turbulent and gives off a roaring sound. This sound is produced by small eddies which develop in the border region between the moving gas and the still air. In a high-pressure steam jet these eddies are very violent and numerous and they produce the familiar hissing sound of such a jet.

When a gas jet is on the point of changing from smooth to turbulent type it is sometimes very sensitive as explained on page 88, *General Physics*. To make a sensitive flame make an assortment of nozzles by drawing short pieces of glass tube to points and cutting off where the inner diameter is about a millimeter. Use a file with its corner ground to a sharp serrated edge, and break off the points clean and square. Trying several nozzles one usually finds one that gives a very sensitive flame with ordinary city gas. City gas pressure is, however, usually not great enough to give the best results, and therefore it is sometimes advisable to fill a small rubber gas bag with the gas and increase the pressure. Natural gas and gas made from gasolene do not burn satisfactorily at a small nozzle. If such gas, only, is available it must be mixed with hydrogen in a gas bag or gasometer. Acetylene gas is not satisfactory because of the smoky character of a large acetylene flame.

36. Instability of the vortex sheet. The spit-ball.—The border region between moving and still fluid is called a vortex sheet.

The instability of the vortex sheet is illustrated by the behavior of the sensitive flame. The behavior of the spit ball as explained on page 89, *General Physics*, is another illustration. Drop a marble in a tall jar of water, or exhibit the zig-zag motion of small bubbles of air as they rise in water.

DISCHARGE RATE OF A STREAM.

In ancient Rome the rate of delivery of water to a household was legally gauged by the size of the delivery pipe. It is evident, however, that a small pipe may deliver water at any rate whatever, depending on the velocity of flow, and commercial water meters as now used in our cities measure the actual amount of water delivered in a given time in cubic feet or in gallons.

The man of the street, or even the man of the farm has a very vague notion indeed of rate of discharge of a stream; his idea is about the same as that which was legalized in ancient Rome. For example, the rate of discharge of a spring is frequently specified as sufficient to fill a one-inch pipe or a two-inch pipe, as the case may be.

Many people, however, seem not to have any idea at all as to a rate of supply of water, as may be illustrated by a recent discussion of the water question in Bethlehem, Pennsylvania, where 2,000,000 gallons per day is needed. Many otherwise intelligent citizens proposed a pipe line from Saylor's Lake, the outflow of which is less than 200,000 gallons per day; there is evidently such a large *quantity* of pure clear water in the lake!

PERMANENT AND VARYING FLOW.

A theoretical discussion of simple lamellar flow of a varying type is given in Part VI of this volume in connection with wave motion in air pipes.

37. The water hammer.—A striking example of varying flow is afforded by the well known water hammer.

When a water wheel is supplied through a long pipe a great drop of pressure occurs at the wheel when the water is turned on and while the water in the pipe is being started (accelerated), and a great rise of pressure, a dangerous rise of pressure, occurs at the wheel when the water-wheel gates are closed. This dangerous rise of pressure is prevented, however, by a relief valve which opens with rise of pressure, or by a stand pipe out of the top of which the water flows when the water-wheel gates are closed.

The consistent use of engineering units. Equations (31), (32), (33), (34) and (35) on pages 93–96, *General Physics*, are true as they stand if foot-slug-second units are used throughout. That is, p must be expressed in "pounds" per square foot, d must be expressed in slugs per cubic foot and so on. See pages 51 and 54 of this volume for further discussion of this point.

38. The disk paradox.—See pages 96–97, *General Physics*. A simple form of this experiment is to blow between the fingers against a small piece of paper lying flat against the palm of the hand.

The mechanics of the disk paradox may be made clear by the following experiment. A jet of water falls centrally upon a thin flat metal disk in a basin of water, and the disk is held up (made to float) by the action of the jet. Everywhere on top of the disk the water has high velocity and low level (low head or pressure), and as the stream of water comes nearly to rest at or near the edge of the disk it raises itself to a higher level (or pressure); whereas the pressure everywhere in the still water underneath the disk corresponds to the high-level water at or near the edge of the disk. A small needle should project upwards from the bottom of the disk and pass through a small hole in the disk to keep the disk from moving sidewise.

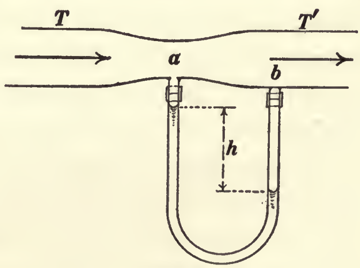

Fig. 22.

39. Diminution of pressure in a throat.—A glass tube TT', Fig. 22, has a throat at a. The tube should be about $\frac{1}{2}$ inch bore and the throat should

be about $\frac{3}{8}$ inch internal diameter. Two short glass nipples are sealed to TT' at a and b, a glass U-tube is connected to these nipples by short pieces of rubber tubing as shown, and a small quantity of colored water is placed in the U-tube.

Blowing through TT' in the direction of the arrow draws the colored water upwards in arm a as indicated in the figure. *The effect of friction, alone, would be to push the colored water downwards in arm* a, because friction alone would cause a lower pressure at b than at a.

This diminution of pressure in a throat is utilized in the jet pump. Water from city mains may, for example, flow through TT' and the pressure at a may be low enough to draw water out of a cellar, the discharge at T' being into the street.

The most striking form of jet pump is the steam boiler injector, and the injector is sometimes operated in practice by exhaust steam. The exhaust steam (because of its low density) attains a very high velocity when it reaches the low-pressure region in the injector throat. Let us assume this velocity to be 1,000 feet per second for the sake of argument, and let us assume that water would gain a velocity of 100 feet per second in issuing as a jet from the boiler (boiler pressure about 75 "pounds" per square inch). Then, neglecting friction, water moving towards the boiler at a velocity of 100 feet per second would carry itself into the boiler. But one unit mass of steam at 1,000 feet per second in mixing with 9 units mass of cold still water gives 100 units mass of solid water moving at a velocity of 100 feet per second, because the initial momentum of the moving steam must be equal to the total momentum of the resultant water, friction against walls of throat being neglected.

40. Ball riding on an air jet.—A light ball like a ping-pong ball is placed over a nozzle from which a blast of air is blowing, and the ball rides in the jet without falling out. One can with the lungs produce a blast sufficiently intense for this experiment. Whenever the ball gets part way out of the moving stream of air the higher pressure of the surrounding still air pushes it back.

41. Ship suction.—See page 98, *General Physics*. The effect of ship suction can be shown by supporting two light balls as shown in Fig. 23 and blowing between them.

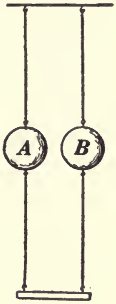

Fig. 23.

The discussion of ship suction in Franklin and MacNutt's *Mechanics* was questioned by a well-known phsyicist, and to settle the matter a letter was written to the Navy Department. The reply, signed by the then Assistant Secretary of the Navy, stated that the effect was unknown to the department. It was added, however, that the attraction of gravitation would pull two ships together! The authors had at the time a dim recollection of the extensive experiments on ship suction that had been carried out with great pains by Naval Constructor (now Admiral) Taylor, and just before receiving the above mentioned reply from the Navy Department one of the authors in a lecture to a group of school teachers in Philadelphia, had found three out of a total twenty-five who knew of ship suction by experience with small boats; and, as a matter of fact, the *attraction* of gravitation between two ships is really a repulsion!! How proud we School Teachers might feel! if it were not largely our fault that an officer of the Navy (for the above mentioned reply was no doubt formulated by a navy officer acting in a clerical capacity) should ever come to substitute the encyclopedia habit for critical knowledge of science, and, also our fault, that silver-spoon youth should never develop beyond hopeless dilettantism, and seldom get even as far as that (for the then Assistant Secretary of the Navy had been a silver-spoon youth with all the tremendous possibilities—and dangers—that go therewith).

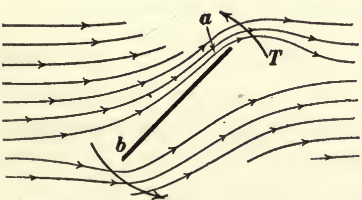

Fig. 24.

42. Flat plate in a moving fluid.—Figure 24 shows the approximate trend of the stream lines of a fluid flowing round a flat

plate *ab*. The fluid velocity is greater at *a* than at *b* so that the pressure of the fluid is greater at *b* than at *a*, and therefore forces are exerted on the plate tending to turn the plate squarely across the stream as indicated by the curved arrows. Drop a small square of moderately stiff writing paper; it turns quickly into a horizontal position and rocks back and forth, and this rocking motion causes the paper to glide back and forth sidewise as it falls. A flat disk like a coin dropped in clear water is easily seen to behave in the same way. A kite with a plane surface tends to rock back and forth, and therefore to dart to and fro sidewise.

A bit of paper with its edges bent upwards falls steadily like a clam shell (with its convex side downwards) in clear water, and a kite with a convex front rides steadily in the air.

Note.—The lines of flow in Fig. 24 represent the flow of an ideal frictionless fluid. In an actual fluid a dead-water region exists behind the plate.

43. Curved flight of a spinning ball.—Cut corn-stalk pith into square pieces and glue them together, thus forming a large block from which make a ball about $1\frac{1}{2}$ inches in diameter. Such a ball thrown from a paste-board tube (a mailing tube) by a quick sweeping motion rolls along the side of the tube and is

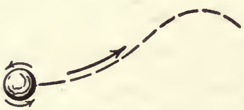

Fig. 25.

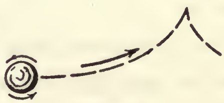

Fig. 26.

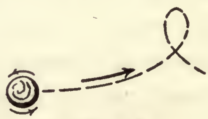

Fig. 27.

set spinning rapidly. A sharply curved flight is the result. Professor P. G. Tait, using a rubber balloon, has succeeded in producing the three theoretically possible "up" curves as shown in Figs. 25, 26 and 27.

44. The curved flight of a high foul-ball.—A ball is set spinning as indicated by the curled arrows *c*, Fig. 28. The catcher is apt to estimate the drop of the ball at *A* as in the case of a thrown

ball not spinning, but because of the rapid spin the high foul turns in towards home base and drops at B. This error of a catcher is sometimes very amusing to watch.

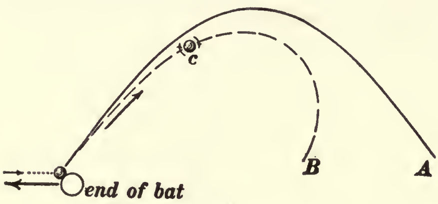

Fig. 28.

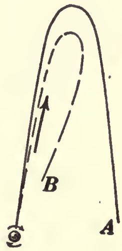

Fig. 29.

The curved flight of a high foul may be shown by shooting a light ball marble fashion so as to give a nearly vertical trajectory as indicated in Fig. 29. A ping pong or pith ball may be used, but the best results can be obtained by using a small oak gall.

45. Viscous friction and eddy friction of fluids.—It is perhaps worth while to demonstrate the two simplest cases of fluid friction as follows: Measure the quantities of water v and v', that are discharged in a given time through a small-bore glass tube, first, with head or pressure p and second with head or pressure p'. Then $v/v' = p/p'$.

Make a pipe or duct consisting of a number of chambers separated by brass disks with comparatively small holes through the disks. Measure the quantities of water v and v' that are discharged through the duct in a given time, first, with head or pressure p and, second, with head or pressure p'. Then $v/v' = \sqrt{p/p'}$.

Remark.—In fairly large pipes eddy friction only is appreciable. At moderate velocities of flow the loss of pressure is very nearly proportional to the square of the velocity. At very high velocities the eddies become more and more fine grained and the loss of pressure increases as v^n, where n is greater than 2.

In the chambered duct as above described the eddies are fixed in location and the loss of pressure is quite accurately proportional to v^2. The increasing fineness of grain of eddy motion with increase of velocity in an ordinary pipe is, in a way, analogous to a closer and closer spacing of the perforated partitions in the above-described chambered duct.

PART II.

HEAT.

OPERATIVE VERSUS INOPERATIVE DEFINITIONS.

Nearly every physical definition, rightly understood, is an actual physical operation. Thus you define a cow pasture by building a fence around it. See page 73 of this volume for further comment.

We think of the flow of an electric current through a wire as being opposed by a kind of frictional drag or resistance very much as the flow of water through a pipe is opposed by a frictional drag. Thus we get the idea of electrical resistance, or, rather, the beginnings of the idea. The rate of generation of heat in a wire is proportional to the square of the current flowing through the wire, *and the proportionality factor has a definite value for the given wire.* Therefore this proportionality factor is adopted as the *measure* of the resistance of the wire.

In every case a preliminary idea of a physical quantity exists before the quantitative definition itself is established, and the quantitative definition, rightly understood, is a group of actual physical operations.

The preliminary idea of electrical resistance is expressed in terms of an analogy and therefore it is easy to grasp and easy to talk about, but the recognition of an idea, or the beginnings of an idea, in mechanics before the quantitative definition is adopted is difficult. For example, John Smith knows that he has gotten something definite when he has secured possession of a given batch of sugar, although he may have no idea as to how much sugar it is. Now the operation of weighing* *always gives the same numerical result when applied to a given batch of sugar* and therefore this result is a most satisfactory measure of quantity of sugar.

The recognition of heat as a measurable quantity depends upon the clear recognition of the fact that if body A is changed thermally by the dissipation of work and brought back to its initial condition by being brought into contact with another (cooler) body B, then the thermal change in B is exactly what would be produced by the dissipation of the original amount of work on body B directly, so that the body A by virtue of the thermal change which is produced in it by the dissipated work "stores" something which is equivalent to the dissipated work. The complex operation thus described is the very essence of the quantitative definition of heat. See page 82 of this volume for further comment.

* An ideal operation of weighing is usually referred to in a brief statement of this kind so as to avoid the necessity of referring to well known sources of error.

HEAT.

Temperature as a condition.—The recognition of temperature as a condition or fact is explained on page 107, *General Physics;* to find a temperature go to a closed cellar or to a stuffy, closed room. A substance not in thermal equilibrium has, strictly speaking, no temperature. Touch a high speed belt or stick a finger into an electric arc and you will be burned, but the degree of hotness cannot be expressed in either case as a temperature. Certain portions of an electric arc, the hot tips of the carbons, for example, seem to be approximately in thermal equilibrium and they can be thought of as having a fairly definite temperature, but it would be necessary to cage a portion of the arc gases, keep them in a heat-insulating enclosure and wait awhile before they could be said to have a definite temperature.

Definite quantitative notions in physics usually depend upon assumed uniformity of distribution in space and time, and to apply such notions to rapidly varying conditions the idea of the limit, as used in the infinitesimal calculus must be introduced. See pages 593–597, *General Physics.* For example the fundamental idea of density refers to a homogeneous substance, and the density of a non-homogeneous substance at a point is defined as the limiting value of $\frac{\Delta m}{\Delta v}$, where Δm is the mass and Δv is the volume of a very small portion of the substance at or near the point.

According to the atomic theory a portion of any substance (even of a substance in thermal equilibrium) departs more and more widely from thermal equilibrium as it is taken smaller and smaller. Therefore the idea of temperature cannot be made to apply to a turbulent substance by using the method of limits. A metal rod which is red hot at one end and cold at the other may, however, be thought of as having a fairly definite temperature at each point because a portion of the material containing many millions of molecules is pretty nearly in the steady condition which we call thermal equilibrium. The idea of temperature is not applicable to a few molecules of any substance unless the average behavior of the few molecules during a long time is taken into consideration.

The atomic-theory conception of temperature.—The atomic theory is a logical structure which is built upon a postulate basis,

and it has a bearing on experimental work and research in that by its means very definite and remarkable ideas can be developed and put to experimental test. Read in this connection pages 119–122, *General Physics*.

The atomic-theory conception of temperature is entirely apart from the things of experiment (entirely apart from thermodynamics), and this conception has been fully developed only for the ideal or perfect gas. See very brief discussion of the atomic theory of gases on pages 325–328, *General Physics*, in particular note the postulate basis as stated on page 325, see the atomic prediction of Boyle's law as involved in equation (ii), page 326, and note on page 328 an assumed conception of temperature which makes our postulated atomic system conform to Gay Lussac's law and to temperature values as measured by the air thermometer.

The principle of the equal-partition of energy (which, by the way, is at variance with many experimental results and cannot, therefore, be true) among the various degrees of freedom of a helter-skelter system seems to indicate that a certain temperature means always a certain average kinetic energy per molecule; but, considering the probable limitation of the principle of equal-partition of energy to the simplest kind of an ideal gas, it is by no means permissible to *define* temperature as the average kinetic energy per molecule of a substance! In fact such a definition is absurd from the laboratory man's point of view, and it would be absurd even if it were entirely legitimate from the point of view of the atomic theory.

Physicists long ago realized the limited usefulness in scientific work of what may, perhaps, be called verbal philosophy. This term unfortunately conveys a suggestion of contempt, but no contempt is here intended, but quite the opposite of contempt, for we refer broadly to that vital blend of intellect and morals which has grown up with the use of words in our age-long dealings with what the philosopher calls human values. Mathematical philosophy is the thing, in the physical sciences. The

idea of continuous quantity with the associated ideas of functional relations and limits was the earliest phase of mathematical physics; later comes the widened use of postulates and of the theoretical structures based thereon; and already there is evidence of a new method (we do not refer to statistical physics) in the mathematics of discrete things, especially in the group theory.

Any carefully ordered set of operations of the analytical chemist is a group, the operations involved in the making of a particular physical measurement constitute a group, and a long step towards realism and adequacy of theoretical physics will be taken when the operations of the physical sciences are connected in a mathematical structure. It is now very difficult to think of these things, but the group theory (or some other kind of mathematics) when fully developed may be expected to make this kind of thinking easy.

So difficult is it to think of the fundamental operations of the physicist that every teacher of physics is more or less indulgent and charitable in his attitude towards the ever recurring recrudescence of verbal philosophy in his field—even when he knows that it is absurd. Mass? It is defined as quantity of matter. Not by any means. It is defined by the operation of weighing by a balance or by the group of operations involved in showing that the unit of mass is accelerated m times as fast as the given body, by a given force; and these operations are simple in comparison with many of the measuring operations in electricity and magnetism. Do you define a cow pasture as a meadow sweet with clover and grass? You do not; you define it, for the cow at least, by building a fence around it!

46. The Brownian motion.—Every student of physics should see the irregular and incessant to-and-fro motion of very fine particles suspended in water, using a good microscope. This motion was discovered by the English botanist, Brown, in 1827, and it is called the *Brownian motion.*

Grind a small amount of insoluble carmine in a few drops of

water, rubbing with the finger in a shallow dish. Place a drop of the mixture on a microscope slide and use a magnifying power of about 400 diameters. The particles in India ink are much finer than the particles of carmine, and a higher magnifying power is required to see them satisfactorily.

47. **The expansion of gases.**—To show the very great expansion of air with rise of temperature place the long stem of a very dry glass bulb in a tall jar of water and heat the bulb. The expansion drives a large amount of air out of the bulb as shown by the bubbles which rise in the tall jar of water. When the bulb cools the contraction of the air which is left in the bulb is shown by the flow of water into the bulb.

The expansion of air, or, rather, its increase of pressure with rise of temperature is utilized in the hot-air engine. A cheap form of hot-air engine is sold by chemical supply houses. In the hot-air engine the contained air is shifted from cold to hot part of cylinder and back again repeatedly by the motion of a large loosely fitting displacement plunger, and the consequent heating and cooling of the air raises and lowers its pressure. The engine piston moves outwards while the air is hot and its pressure is high, and the energy of the fly wheel then pushes the piston inwards and compresses the air while it is cold and its pressure is low. A wooden model showing the large displacement plunger, the long cylinder and the piston, and showing the red hot part of the cylinder painted red is a great help in the explanation of the action of the hot-air engine.

The expansion of a gas, or rather its rise of pressure with rise of temperature is also utilized in the gas engine. In this case, however, the gas which is used is a mixture of inflammable gas or vapor and air, and the rise of temperature is produced by the quick burning of the mixture. The toy gas cannon is the same in principle as the gas engine and it makes a good piece of demonstration apparatus.

Concerning the gas equation.—Several years ago a physics teaching friend sent us the following example of what he con-

sidered to be bad-medicine, and we agree with him. To fix the formula $L = \frac{1}{2}at^2$ for the distance L traveled by a falling body in t seconds the student is asked to remember that L is in fact one half of a tee-square as indicated in Fig. 30.

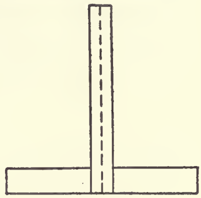

Fig. 30.
L is one half of a tee-square.

The unintelligent use of formulas is certainly a bad habit, and the following example is taken from our own personal experience. It concerns the familiar gas equation $pv = RT$. A physics teacher walked to the University one morning in company with an engineering instructor, and the conversation turned to the gas turbine, concerning which the engineering instructor had some new ideas which he hoped to develop practically. After some discussion it appeared that the engineering instructor was perplexed concerning a theoretical phase (as he would have expressed it) of gas action, and it soon became clearly evident that the perplexity was this: "Since $pv = RT$, how can a gas at high pressure be cold?" The young man did not recognize the two variables p and v, and he did not have the faintest appreciation of the fact that this equation applies only to a given amount of a gas. He had allowed the formula to take the place of the simplest kind of common sense, namely, that anything may be as hot or as cold as one may care to make it. A verse from one of the *Bab Ballads* comes, willy-nilly, into our minds as we relate this actual occurrence:

"Oh, list to this incredible tale
Of Thomson Green and Harriet Hale
Its truth in one remark you'll sum;
Twaddle, twaddle, twaddle; twaddle twum."

Of course this engineering instructor had taken the regulation dose of engineering thermodynamics and had passed it, for no one can become an engineering instructor who has not graduated from a technical school.

"Es erben sich Gesetz und Rechte wie eine ew'ge Krankheit fort."

and what a privilege (Recht) it is to become an engineering instructor!

Nernst has said that by far the most useful theoretical developments in thermodynamics are those which are expressed explicitly in terms of physical operations, such as Carnot cycle operations; and there are many other operations of equal importance. It is doubtful if any experimental research in chemistry or physics or engineering has ever been suggested by the elaborate algebraic developments to which thermodynamic theory unfortunately lends itself.

A perfectly clear understanding of the physical meaning of the second law of thermodynamics (which *is* possible because of the postulate character of the law) and ability to interpret an argument in thermodynamics in terms of definite physical operations, these are the all-important things; and no engineer, however great his pretended predilection for practicality, can ever arrive at these all-important things when he starts out with a purely algebraic guess "that heat is expressible as the product of two factors, one of which is entropy" and remains satisfied with his algebraic guess as a definition of entropy!

All there is to engineering thermodynamics, *and a great deal more*, is included in pages 105–177, *General Physics*, supplemented by Marks and Davis's *Steam Tables*. It may be that we should include, as an essential supplement, the presumably unpublished results of a recent research concerning the temperature-entropy diagram for which an unnamed manufacturing concern is said to have appropriated several hundred dollars.* Not knowing the results of this investigation, however, we are inclined to point out what would seem to us to be the only proper line of "research" in this matter. Send a young man to, let us say, Berlin to take a course under Nernst and Planck. This line of research is the

* See a reply by a Dean of Engineering to our statement as to the limited usefulness of the temperature-entropy diagram. *Bulletin of the Society for the Promotion of Engineering Education*, January 1917, pages 270–274. Our statement is to be found in the same *Bulletin*, November 1915 pages 168–190.

cheapest possible in this particular case, it would lead in three years to perfectly definite results, and on the Cecil Rhodes scale it would cost precisely 4,500 dollars. It is not often that one is in a position to make these three definite specifications regarding research; and also, alas, it is not very often that a manufacturing concern is willing to appropriate even a small fraction of the necessary cost of a piece of research!

48. Northrup's model of a gas.—A glass jar containing many thousands of small steel balls has a rapidly moving element or agitator at the bottom which keeps the balls flying about, and the mouth of the jar is covered by a disk which is freely suspended from a balance arm. The flying balls in the jar exhibit the properties of a gas and conform to Boyle's law and to Gay Lussac's law* as follows:

A given speed of the agitator corresponds to a definite average kinetic energy per ball, and doubling the speed of the agitator quadruples the average kinetic energy per ball. Therefore square of speed of agitator corresponds to or is proportional to Kelvin temperature T.

(*a*) *Boyle's law.*—Speed of agitator being kept constant, the force exerted on the glass disk is found by test to be inversely proportional to the volume of the space in which the balls fly about (pressure inversely proportional to volume at constant temperature).

(*b*) *Gay Lussac's law as relating to change of pressure at constant volume.*—The force exerted on the glass disk is found by test to be proportional to the square of the speed of the agitator, volume of space in which balls fly about being constant (pressure proportional to T at constant volume).

(*c*) *Gay Lussac's law as relating to change of volume at constant pressure.*—The force exerted on the glass disk is found by test to be unchanged in value when the square of the speed of the agitator and the volume of the space in which the balls fly

* We refer here particularly to Gay Lussac's Law as stated in Art. 75, page 113, *General Physics.*

about are both doubled (volume is proportional to T at constant pressure).

(*d*) *The Brownian motion.*—A steel or wooden ball much larger than the small steel balls or "molecules" is suspended in the vessel and it moves erratically to and fro and up and down as the small balls strike against it. A difficulty in this experiment is that the larger ball must be suspended by a string, it tends to vibrate to and fro regularly as a pendulum, and this regular pendulous motion is mixed up with the erratic motion (the Brownian motion).

49. **Expansion of liquids and solids.**—The relative expansion of water and glass may be shown by heating a glass bulb filled with colored water which rises in a narrow glass tube. It is most satisfactory to place this stem in the field of the projection lantern. When the heating flame is brought against the bulb the mean temperature of the glass walls is suddenly raised and the sudden expansion of the glass is shown by a quick drop of the water level in the stem. Then continued heating warms both the water and the glass, and the greater expansion of the water shows itself by a slow rise of water level in the stem. Very few students seem to appreciate the fact that water contracts as it is warmed from 0° C. to 4° C., and it is worth while, therefore, to repeat this experiment starting with water bulb at 0° C.

The irregularity of expansion of steel in the neighborhood of 700° C. may very properly be shown at this point and the phenomenon of recalescence referred to later. See Experiment 66.

Several samples of good chemical thermometers should be exhibited, and it will do no harm to state that a thermometer *bulb and stem* should be completely immersed in a fluid or region in order that it may indicate the temperature of the region accurately. In many cases, however, the stem must be allowed to project into the outside air to make it possible to take a reading. The authors know of a junior student in engineering who removed

a thermometer from a calorimeter vessel and held it up to a light so as to be able to read it accurately!

50. The Trevelyan rocker.—A moderately hot soldering-copper is laid on block of lead resting on the table, and, once started, the copper continues to rock and produce a buzzing sound. At each momentary contact of the rocking copper with the lead, the lead near the point of contact is heated and the expansion of the lead (a local swelling too small to see) gives a push to the receding copper and thus keeps up the rocking motion. The lead should present a cleanly cut surface where the copper comes into contact with it, and the copper itself should be scraped clean. With a little patience this curious experiment can always be made to work.

Fig. 31.

51. Shrinking bar and cast iron pin.—A steel bar *bb*, Fig. 31, has a large pin *P* fixed to it permanently, and a removable cast iron pin *p*. The middle part of the bar *bb* is heated nearly to a red heat, the pin *p* is put in place, and the bar *bb* is then lowered into position between two lugs *l* and two lugs *l′* as indicated. The bar *bb* contracts as it cools, and breaks the pin *p*.

52. Shrinking of collar on a shaft.—The important shop operation of shrinking a collar on a shaft makes a good lecture demonstration. A steel collar turned to 1.99 inch inside diameter and heated can be slipped over a 2-inch shaft if one does it quickly so that the hot collar has no time to cool.

53. Thermo-stresses.—The thin parts of an iron casting cool quickly and contract, and as the thicker parts of the casting then cool and contract excessive stresses are produced which often result in the fracture of the casting.

When a pane of glass is heated quickly over a flame the under surface which is in contact with the flame heats and expands, the upper portions of the pane are subjected to excessive tension,

and the pane breaks. Holding the pane in the hand one experiences a fairly severe shock due to the quick outward movement which occurs at the instant of rupture. It is not entirely safe to try this experiment.

The most interesting example of thermo-stresses is furnished by the well-known Prince Rupert's drops. Hold the end of a small glass rod in a blowpipe flame and allow the end of the rod as it drops to fall into water. Most of the drops thus formed explode before one can take them out of the water; but by trying patiently and especially by trying several varieties of glass one can get drops which stand indefinitely, but which explode suddenly when the surface is scratched or when the long tail is broken off. One can feel a very decided shock when a drop explodes between the fingers. This is a perfectly safe experiment if one closes one's eyes. A drop may be held in the field of the lantern and it suddenly disappears on explosion. The most satisfactory proceeding is to place a drop on a horizontal flat glass plate in the lantern (vertical projection arrangement), and break off the tail of the drop by means of a small pair of pliers.

Fused quartz is remarkable in that its contraction when cooled from a high red heat is not enough to produce in it a stress sufficiently great to produce rupture. A clear quartz tube may be heated in a blast lamp and quenched in water without breaking.

Heating by compression. Cooling by expansion.—Any substance which expands with rise of temperature has its temperature raised by compression or lowered by expansion. This effect is especially great in the case of a gas.

54. The fire syringe and Diesel-engine ignition.—A small plunger fits in a small cylinder and to the plunger is attached a bit of dry cotton cloth. The air in the cylinder is compressed by pushing the plunger in very quickly, and the cotton is found to be burning if the plunger is quickly withdrawn. The air in the cylinder is heated by the compression and the cotton is ignited. This method of ignition is used in the Diesel engine.

The moving piston draws in a cylinder full of fresh air which is then compressed by the energy of the flywheel and heated so that an oil spray injected into the compressed air is ignited and the burning of the oil keeps the temperature (and pressure) high as the piston recedes.

55. The formation of cumulous clouds.—On a quiet, warm summer's day the sun-heated air near the ground frequently rises in chimney-like columns like *aa*, Fig. 32. As the air rises its pressure decreases and it expands. As it expands its temperature falls; and at a certain level *ll* the falling temperature reaches the dew point and the water vapor in the air begins to condense and form cloud or fog. The beautiful dome-like clouds so commonly seen on a quiet summer's day are the fog-filled tops of such rising columns of warm moist air, and a striking characteristic of these clouds is their level bottoms.

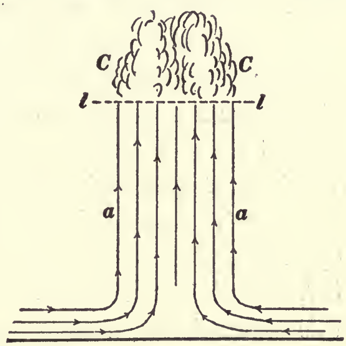

Fig. 32.

This cooling of air by expansion can be shown as follows: A large glass flask containing a little water is warmed slightly over a flame so that the contained air is warm and moist. Applying the mouth to the flask a portion of the air can be sucked out (causing a decrease of pressure) and allowed to flow back in again (causing a rise of pressure). At each drop of pressure the air in the flask is cooled and the flask is filled with cloud. At each rise of pressure the air in the flask is warmed slightly and the fog or cloud disappears.

THE CONSERVATION OF ENERGY AND THE FIRST LAW OF THERMODYNAMICS.

Any definition or principle in physics should be stated so as to call to mind the operation upon which the definition or principle is based, or, in other words, to suggest trial and verification. Tell a woman she cannot drive a nail and she will, if she takes you seriously, get hammer and nail and try. When Tom Sawyer tells his playmates that they cannot whitewash a fence they are all eagerness to submit the proposition to an experimental test. But what young man ever dreamed of trial and verification when told that energy can neither be created nor destroyed? Indeed the statement does not suggest any physical operation whatever, it is actually meaningless unless one knows beforehand just what its meaning is intended to be. Read in this connection Art. 48, pages 68–70, *General Physics*.

The extension of the principle of the conservation of energy so as to cover heat effects (including all chemical effects) is called the first law of thermodynamics, and stated in the language of experiment, it is as follows: *A certain change of state is produced in a substance A by the dissipation of work, and the substance A is brought back to its initial condition by contact with another cooler in substance B; then the change produced B is exactly the same as if the original work had been expended (dissipated) on B directly.* How much more intelligible this statement than any abstract statement concerning the equivalence of work and heat!

Take a coin and rub it on a board, pretending to heat only the coin, then place the coin in contact with a colder object as if to bring the coin back to its initial condition, and point out that the change which would thus be produced in the colder object would be exactly the same as if the original work had been expended on the colder object directly.

The molecular conception of heat.—When heat is imparted to a substance the temperature of the substance usually rises, but not in every case. Thus when heat is imparted to ice at 0° C. the ice is converted into water at the same temperature. It may be that such a change as the melting of ice increases the number of free molecules so that in spite of a constant average kinetic energy per molecule (constant temperature) the whole of the imparted heat goes to increase the kinetic energy of molecular motion. If this were true in general it would be permissible to speak of heat as the kinetic energy of molecular motion; but it is not true in general and therefore when heat is imparted to a substance it is stored in the substance partly as an increase of molecular kinetic energy and partly, no doubt, as an increase of molecular potential energy due to a change of molecular configuration of the substance.

The only case where heat imparted to a substance is known to be stored solely as increased molecular kinetic energy is in the "ideal" or "perfect" gas which does not change its temperature during free expansion. This matter is discussed very briefly on page 170, *General Physics*, and the important conception of the ideal or perfect gas as a gas which conforms to Boyle's law and does not change its temperature during free expansion is explained on pages 171–175, *General Physics*. Free expansion is discussed more at length on pages 344–347 of Franklin and MacNutt's *Mechanics and Heat.*

Heat of combustion.—When we ask a young man in class to give a definition we always insist—or try to insist, which, alas is not the same thing—on a reply which clearly suggests or definitely postulates the physical operation to which the definition relates and in which it finds its meaning. Thus we would wish a young man to say in defining the heat of combustion of coal that it is the amount of heat you get out of a pound of coal when you burn it. A freshman engineer was asked to give this definition; he had learned before to avoid the use of the word *per* in the intimate and more or less informal talk of the classroom

and he managed to get all of the definition clearly stated except to say that the coal was to be burned. This he could not be led to say, and finally, being asked bluntly how to get heat out of coal he replied "Why, professor, I don't know!"

This young man evidently did not believe that the study of a mathematical science in a college (for even our technical students "go to college" if you please) could possibly relate to so familiar a thing as the burning of coal, and we could not blame him. For, months before the question as to how to get heat out of coal was put to him he had been studying calculus under a class-room regime which had given from 40 to 60 percent of failures year after year, and the unintelligibility of the text book which was used may be fairly judged by the following which is given as an example in a two-page discussion of discontinuity (pages 22 and 23 in fact): —

"Another form of discontinuity is seen in the function $y = \dfrac{2^{1/x} + 2}{2^{1/x} + 1}$ when $x = 0$. Here y approaches two limits, according as x approaches zero through positive or negative values.

$$\operatorname*{Lim}_{x=+0} \frac{2^{1/x} + 2}{2^{1/x} + 1} = 1. \quad \operatorname*{Lim}_{x=-0} \frac{2^{1/x} + 2}{2^{1/x} + 1} = 2.$$

We see that when $x = 0$ the curve jumps from $y = 2$ to $y = 1$, that is from B to A [referring to a figure]."

When will our mathematics teachers learn that to be exacting *and* unintelligible is fatal? When will they learn to appreciate the tremendous importance of appealing to the intuition or sense-complex of a student, *especially in a brief and necessarily incomplete introductory discussion of an idea?* Any young man can "see" a poor trapped sparrow butt his head against a window pane, and a notion of discontinuity based upon such is incomparably more useful even in the study of calculus than any purely formal notion, because the young man does not "see" such things.

Transfer of heat.—The three processes by which heat is transferred, namely, conduction, radiation and convection, are very briefly referred to on page 131, *General Physics*, and the elementary theory of heat conduction is there outlined.

An elementary presentation of the subject of radiation from the point of view of the theory of heat (involving both the atomic theory and thermodynamics) is given on pages 301–316 of Franklin and MacNutt's *Light and Sound*, The Macmillan Co., 1909.

The transfer of heat by convection is exemplified in the heating of buildings, and the purely empirical rules and equations which are used by heating engineers are to be found on pages 653–687 of Kent's Handbook.

56. Safety lamp experiment.—A horizontal piece of wire gauze is lowered quickly into a flame, then for a few moments the gauze conducts heat away from the flame and cools the gases to such an extent that combustion ceases above the gauze. The flame stops at the gauze. Turn on a Bunsen burner, place a piece of wire gauze an inch or two above the burner, light the gas above the gauze, and the flame burns steadily above the gauze without striking through the gauze. This action of wire gauze is utilized in the well-known miner's safety lamp, which was invented by Sir Humphrey Davy.

57. Cloth singeing experiment.—Draw a piece of fuzzy cloth tightly around a metal cylinder (a short piece of steel shafting) and run the flame of a Bunsen burner quickly over the cloth. The body of the cloth is kept cool by contact with the metal, but the fine fuzz is singed off. In the finishing of fine glossy silk the cloth is run over a flame in close contact with a cool metal cylinder.

Wrap a thin sheet of paper around a cylinder of which part is solid metal and part is wood. Hold the paper-covered cylinder in the flame of a Bunsen burner for a few moments. The paper which is backed by wood will be badly scorched whereas the paper which is backed by metal will not be perceptibly scorched.

Heat insulation.—The most nearly complete heat insulation is that of the Dewar bulb or thermos bottle, the action of which is explained on pages 310–311 of Franklin and MacNutt's *Light and Sound.*

Porous heat-insulating materials, such as saw-dust, wool, cork-board, hair felt and magnesia pipe covering owe their heat-insulating properties to the low heat conductivity of the air which lies stagnant in the pores of the material.

The phase rule.—The discussion of the phase rule as given on pages 107–108, *General Physics*, should perhaps be given in Chapter VIII.

58. Crystallization experiments.—(*a*) Moisten a very clean glass plate with a solution of ammonium chloride and place it in the lantern.

(*b*) Make a hot concentrated solution of potassium chlorate, and allow it to cool with occasional brisk stirring. After each stirring great numbers of minute crystals form, and as these crystals grow they settle towards the bottom as beautiful thin plates, all alike. These thin plates show beautiful colors if the vessel is placed in sunlight. The cooling solution can be placed in a lantern cell and the floating crystals projected on the screen.

Segregation.—If a vessel of dilute salt solution is frozen, nearly pure ice forms at first, nearly the whole of the salt accumulates in the residual mother liquor at the center, and this residual liquor then freezes if the temperature is sufficiently lowered. Something similar to this generally takes place when any liquid mixture is frozen. Thus certain of the impurities in cast iron or steel collect near the axis of an ingot. It is advisable to exhibit a cut and polished section of a small ingot of steel or cast iron or brass or lead-antimony alloy. A small crucible full of the alloy is allowed to cool slowly and the resulting ingot cut and polished. The visible difference between core and rim of ingot is increased by etching.

59. The boiling paradox.—A round-bottomed flask with tightly fitting stopper and a 30-inch stem contains water, it is boiled until all of the air is driven off, and then quickly inverted in a cup of mercury which stands in a deep tray or tub. See Fig. 33.

Pouring cold water on the upturned bottom of the flask, reduces the pressure momentarily, as indicated by the mercury column, and causes the water to boil.

Fig. 33.

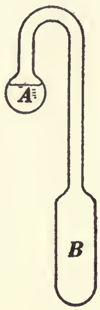

Fig. 34.

60. The cryophorus.—The two connected bulbs *A* and *B*, Fig. 34, contain nothing but water (in *A*) and water vapor. Place bulb *B* in a mixture of ice and salt, and the water in *A* freezes. The water vapor pressure is kept at a very low value by the cooling of *B*, and the vaporization of the water in *A* reduces its temperature sufficiently to freeze it. Bulb *B* should be stirred round and round in the ice and salt, and bulb *A* should be occasionally given a quick blow with the hand, otherwise the water in *A* may be cooled far below the freezing point without freezing.

61. The bursting of a cast-iron ball by freezing water.—A cast-iron ball 4 or 5 inches in diameter with walls about $\frac{1}{2}$ inch thick is filled full of water, plugged with a paraffine- or wax-covered screw, and placed in a mixture of ice and salt.

A curious fact is that a pipe filled with boiled (air-free) water is much more likely to burst on freezing than a pipe filled with fresh water (containing air in solution). As the fresh water freezes the air is segregated, a line of air bubbles is left along the axis of the pipe, and this line of air bubbles may act as a duct to relieve the pressure in a farther portion of the pipe. The bubbles in ordinary "artificial ice" and most of the bubbles in pond ice are the air which is segregated during the freezing.

62. **Regelation.**—A loop of fine steel wire with two heavy weights is hung over a block of ice, and in the course of an hour the wire will cut through the ice, but leave the block as one piece. In front of the wire where the pressure is great the freezing point (or melting point) of the ice is low, say, $-1°$ C. Therefore some of the ice (originally at 0° C.) in front of the wire melts and the temperature of the region in front of the wire falls to $-1°$ C. and stands at that temperature. The water formed by the melting flows to the back of the wire where it is free from excess pressure and where it stands at 0° C. Therefore heat flows steadily across the wire from the warm region back of the wire to the cooler region in front of the wire, the water back of the wire is continuously frozen by this loss of heat, and the ice in front of the wire is continuously melted by this supply of heat.

63. **Wood's metal.**—A striking example of low-melting-point alloy is Wood's metal which consists of bismuth 50.1 %, cadmium 10.8 %, lead 24.9% and tin 14.2 %. Its melting point is 65.5° C., whereas the most fusible of its constituents is tin with a melting point of 232° C.

Select a heavy-pattern teaspoon, imbed it in wax, cut away the wax even with one face of the spoon, and cover spoon and wax with a block of plaster of Paris. When the plaster block is hard lift it off, varnish or oil its face, lay the spoon upon it and cover with a second block of plaster. When this second block of plaster is hard lift it off, loosen and remove the spoon and cut

a gate through which Wood's metal may be poured into the mould.

64. Bottle washer's experiment.—A large dry bottle with a tightly fitting stopper is warmed carefully over a flame, a small sealed glass bulb of water is placed in the bottle, the bottle is stopped, and the bulb is broken by shaking the bottle. If the bottle is at 50° C. the pressure in the bottle rises to 92 millimeters above the atmosphere, and a decided burst of spray is produced when the stopper is removed.

Transformation points or temperatures.—Many solid substances can exist in several states or modifications as in the case of ice as briefly described on page 147, *General Physics*, the change from one state or modification to another state or modification takes place at a definite *transformation temperature* (at a given pressure), and the change is generally accompanied by the evolution of heat (as in the freezing of water) or the absorption of heat (as in the melting of ice).

This matter may be best exemplified by a detailed study in the laboratory. An ounce or two of ammonium nitrate crystals is melted in a small glass beaker, a thermometer reading up to 200° C. is placed in the melt, the heating flame is removed and the thermometer readings are taken at intervals of 30 seconds while the beaker cools. The melt freezes at 168° C., and changes its crystalline structure at 125° C., again at 85° C., and again at 35° C.

Retarded transformations.—In many cases a pure substance in one state or modification can be heated above or cooled below the transformation temperature at which it normally changes to another modification before the actual change takes place. See pages 151–152, *General Physics*.

65. Superheating and undercooling of water.—Heat a small quantity of very pure air-free water in a clean test tube. The water will rise very considerably above its normal boiling point,

and when boiling does begin it takes place with almost explosive violence, throwing most of the water out of the tube. This experiment succeeds best if a small amount of sulphuric acid is added to the water. In this case the normal boiling point is raised, but the dilute acid does not boil perceptibly until it is heated above its normal boiling point, as stated. The water is freed from air by previous boiling in a larger vessel like a flask.

Cool a small quantity of very pure air-free water in a clean glass flask by dipping the flask repeatedly into a mixture of ice and salt. The water cools 5 or 6 degrees below 0° C. (as may be indicated by a thermometer with a very clean bulb) without freezing, and then a sudden shock will cause almost instant formation of fine ice crystals throughout the water.

To cool water 5° or 6° below 0° C. without freezing, the water must be air free and extremely clean. It is difficult to keep the water clean enough in an open vessel. Therefore this experiment succeeds best by using an ordinary water hammer which contains clean boiled water. Dip the containing bulb into slightly salt water containing pieces of ice (temperature of mixture about $-5°$ C.), remove carefully and shake. Instead of a sealed water hammer one may boil distilled water in a clean flask and close the flask with a clean rubber stopper while the boiling is taking place, and a clean-bulb thermometer may pass through the stopper to indicate the temperature of the undercooled water in the flask.

Superheated and undercooled water are examples of retarded transformations. Also a super-saturated solution of a salt is an example of a retarded transformation.

Make a hot saturated solution of sodium acetate in a small clean flask, stop the flask with a clean cork and set it aside to cool. The cool supersaturated solution crystallizes suddenly when a small crystal of sodium acetate is dropped into the flask or when the solution is touched by a stick which has been wet with sodium acetate solution and allowed to dry.

This experiment may be projected by the lantern by using a small flask and placing it in a water filled lantern cell.

An example of an almost permanently retarded transformation is as follows: Make a strong syrup by dissolving granulated sugar (or, better, white rock candy) in a small quantity of hot water, boil the syrup over a slow fire until nearly all the water is driven off, and set aside to cool. The solution does not freeze (crystallize) at once but cools to a glass-like substance. Place a small crystal of granulated sugar on the surface of the glass-like substance, press it into close contact therewith, and allow the whole to stand in a cool place for several days. A crystalline growth will slowly spread out from the crystal of granulated sugar, showing the slow transformation of the glass-like substance (which is the stable form at higher temperature) to a crystalline form (which is the stable form at lower temperature).

Pure sugar cannot be melted, because it is decomposed chemically before its true melting point is reached, but nearly all of the water in syrup can be driven off, leaving the pure or nearly pure sugar in the melted condition (as a retarded state or condition).

66. The recalescence of steel.—A piano steel wire about 0.04 inch in diameter and 5 or 6 feet long is arranged as indicated in Fig. 35, the terminals *a* and *b* being connected to 110-volt supply mains through a suitable rheostat and a controlling switch. Two small plates are clamped to the wire at the lower end, and a light wooden pointer catches under a cleat at *c*, straddles the wire and rests on the small clamped plates as indicated.

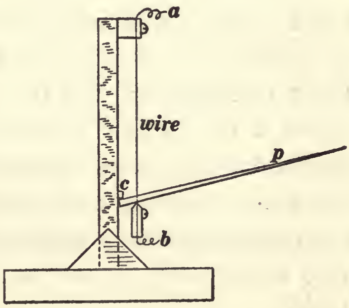

Fig. 35.

The movement of the pointer while the wire is being heated or cooled shows a marked irregu-

larity of expansion at the transition temperature, which is about 700° C.

As the wire is being heated its temperature rises a little above 700° C. before the change of state or condition takes place, and then when the change does take place a very noticeable momentary drop in temperature (decrease of brightness of the hot wire) occurs.

As the wire is being cooled its temperature falls a little below 700° C. before the change of state takes place, and then when the change does take place a very noticeable momentary rise of temperature occurs.

67. Hardening and tempering of steel.—Heat a small rod of tool steel to bright redness, quench it in water, and show that it will scratch glass and that it is extremely brittle. Explain the tempering process as a more or less complete conversion of the hardened steel to the condition which is stable at low temperatures (soft variety of steel). This conversion takes place at ordinary room temperature but it is greatly hastened by rise of temperature. A given degree of tempering is produced by keeping a glass hard piece of steel at a certain temperature for a certain time, and the higher the temperature the shorter the time required for a given degree of tempering.

Make the points of three steel tools glass hard, then make small polished areas at tips of tools, hold tools one at a time with portions just back of tips in a Bunsen flame, watch the polished areas carefully, quench the first tool when the polished area shows a light straw color, quench the second tool when the polished area shows a brownish color, and quench the third tool when the polished area shows a brilliant blue color. These colors depend upon a combination of time and temperature and they are extensively used as indicators in the tempering of steel tools.

Diffusion and osmosis.—When a solution of salt is in thermal equilibrium it is homogeneous, that is to say, it has the same

degree of concentration everywhere. When a solution of salt is more concentrated at the bottom of a vessel, for example, than at the top, the inequalities of concentration are slowly eliminated by the movement, the very slow movement, of the dissolved salt through the solution from the region of high concentration to the region of low concentration. This movement of the dissolved salt is called *diffusion.*

When solutions of different concentration are separated by a thin membrane, the diffusion of the salt through the membrane is hindered more or less, whereas the water passes through more freely. A thin bladder which is filled with water, tied and placed in a vessel of brine will become flabby because of the loss of water; and if the bladder is partly filled with brine, tied and placed in a vessel of water it will swell until it bursts because of the entrance of water. If the brine-filled bladder does not burst the pressure in it rises to a fairly definite value, and this limiting pressure, reckoned above the pressure of the surrounding water, is called the *osmotic pressure* of the brine, and the action of the membrane in permitting selective diffusion is called *osmosis*. This phenomenon of osmosis which is a very widely occurring phenomenon, has given rise to the precise conception of the *semi-permeable membrane*, a conception which is of great utility in the theory of solutions.

Sprinkle salt on fresh meat or fish, and the meat or fish appears to become wet. Each individual cell of the meat or fish behaves like a bladder filled with water (which indeed it is) and placed in strong brine. Water flows out of the cells through the cell walls.

68. Solution and diffusion of $CuSO_4$.—Drop a few crystals into a tall jar of water and place the jar where the class can see from day to day the slow diffusion of the copper sulphate upwards.

69. Osmosis.—Fill a bladder partly full of strong brine, tie the bladder, place it in a jar of water, and place the jar where the slow distension of the bladder can be seen.

The second law of thermodynamics.—Next to the principle of the conservation of energy the most important generalization of physics is the second law of thermodynamics or the principle of entropy, as it is sometimes called, and no one can have any sort of grasp of the philosophy of physics who does not have some degree of understanding of both of these generalizations. The student should, therefore, be required to study very carefully pages 152–169, *General Physics.*

70. Thermo-elastic properties of rubber.—Cooling by expansion and heating by compression is usually very small in liquids and solids and difficult to detect. Indeed this is the case with ordinary rubber insofar as change of volume due to change of pressure is concerned. But when a rubber band is stretched it is very perceptibly cooled, and when the stretch is relieved the band is warmed. This may be shown by stretching and shortening a rubber band repeatedly and holding the band in contact with the lips.

An interesting example of a widely applicable argument based on the second law of thermodynamics is given on pages 393–396 of Franklin and MacNutt's *Mechanics and Heat.* Knowing that a rubber band cools when stretched, this argument shows that a rubber band stretched by a hanging weight must shorten when heated; and this conclusion is easily verified by experiment.

PART III

ELECTRICITY AND MAGNETISM

THE SIDE-STEPPING OF MATHEMATICS.

The discussion of Pascal's principle on pages 52 and 53 of this volume illustrates a wide-spread tendency among teachers of physics, a contempt for precise thinking, which is deplorable. What are we to do? And the mathematics teacher also shows his contempt for mathematics when he side-steps mathematical ideas and pins his faith—and his students—to formalism and machinery. What are we to do? Our own inclination is that of P. G. Tait who in 1876 wrote that "In defense of accuracy we must be zealous, as it were, even to slaying." But we thank Professor Tait for the humorous turn in his qualifying clause; for indeed no human being, and least of all a timid teacher, would wish in these terrible times to do no more than run amuck, as it were, among his friends. See pages 53 and 106 of this volume for further comment.

MAGNETISM AND THE ELECTRIC CURRENT.

The traditional plan in the elementary treatment of electricity and magnetism is to begin with electrostatics, but the authors believe that it is much better to begin with magnetism and the electric current because in this plan a wide range of simply related experimental facts comes quickly into view; the many-sided magnetic effect of the electric current (see pages 181, 196, 198, 199, 209, 254 and 276, *General Physics*), the chemical effect of the electric current, and the heating effect of the electric current.

ONE SET OF ELECTRICAL UNITS IS SUFFICIENT.

It is a fact, however reluctant some of our physicists may be to admit it, that *one set of units of measurement is sufficient.* We make exclusive use, therefore, of the units of the so-called *"electromagnetic" system.*

Some familiarity with the "electrostatic" system of units is demanded, however, by the wide use of this system in the literature of electricity and magnetism, but essential demands and conventional demands are not on the same plane, and the authors emphasize this fact by ignoring the "electrostatic" system of units entirely. It is sufficient to present the main features of the electrostatic system of units orally in class.

THE MYSTERIOUS ERUPTION OF "V" IN THE DISCUSSION OF THE TWO SYSTEMS OF ELECTRICAL UNITS.

Any discussion which places emphasis on the fact that the "electromagnetic" unit of charge divided by the "electrostatic" unit of charge is equal to the square of the velocity of light, but which stops short of a complete elementary discussion of electromagnetic wave motion, is, in our opinion, misleading and fantastic. Suppose we had two rival systems of mechanical units, one of which was based on the arbitrary assignment of unity as the density of the air and the other on the arbitrary assignment of unity as the compressibility of air, and suppose that the only reference to sound in our whole treatment of physics were to mention the mysterious eruption of v (the velocity of sound) in the ratios of the units in the two systems! would such a thing make for clear understanding? It certainly would not. We have, for this reason, chosen to treat the factor B in equation (i), page 293, *General Physics*, as a simple proportionality factor to be determined by experiment, and it is exactly that. Indeed it is nothing more than that to one who does not go into the theory of electromagnetic wave motion.

ELECTROMAGNETIC WAVE MOTION.*

Electrical effect of a magnetic field which moves in a direction at right angles to itself; sidewise motion of magnetic lines of force.—The electromotive force along a path which encircles an amount of magnetic flux Φ is

$$E = -\frac{d\Phi}{dt} \tag{1}$$

where E is expressed in abvolts and $\frac{d\Phi}{dt}$ is the rate of change of the magnetic flux. See equation (81), page 256, *General Physics*. The meaning of the negative sign is that the induced electromotive force is in the direction around the path (or circuit) in which a *left-handed screw* would have to be turned to travel in the direction in which the increasing flux Φ passes through the opening of the circuit (in the direction of the force with which the magnetic field which produces Φ would act on a north pointing magnet pole).

Another form of equation (1) is

$$E = lHv \tag{2}$$

where E is the electromotive force in abvolts induced in a straight rod l centimeters long which travels sidewise at a velocity of v centimeters per second across a magnetic field of which the intensity is H gausses as indicated in Fig. 184, page 255, *General Physics*. See equation (80), page 255, *General Physics*.

Let us think of the rod bc, Fig. 184, page 255, *General Physics*, as stationary and imagine the magnetic field as sweeping past

* This discussion is not intended to be entirely rigorous, or complete. Indeed the idea of sidewise motion of identifiable lines of force is highly artificial. A better point of view is that in which the differential equations of wave motion are derived from the fundamental circuit-equations (1) and (8) below. This point of view is developed in Part VI of this volume.

the rod at velocity equal and opposite to v. Then equation (2) still applies, E is the electromotive force in abvolts along the stationary rod, and this electromotive force exists whether the rod is there or not so that E/l *is the electric field intensity in abvolts per centimeter produced by the sidewise motion of the magnetic field.* Therefore, representing this electric field intensity by f and remembering that v has been reversed in sign, equation (2) becomes

$$f = -vH \tag{3}$$

Magnetic effect of an electric field which moves in a direction at right angles to itself; sidewise motion of electric lines of force.—A charged condenser AA BB, Fig. 36, is allowed to

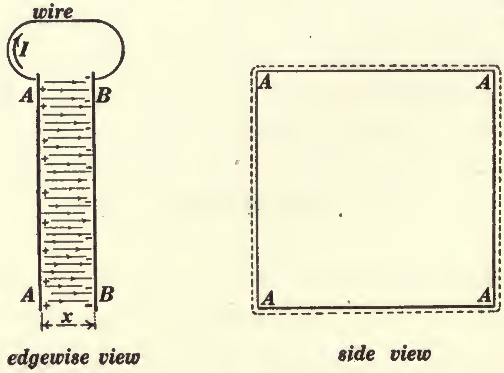

Fig. 36.

discharge through a wire. The current I is equal to the rate of decrease dQ/dt of the charge on either plate. But $Q = CE$, where E is the electromotive force between the condenser plates in abvolts and C is the capacity of the condenser in abfarads, and

$$C = B\frac{a}{x} \tag{4}$$

where a is the area of one face of a condenser plate, x is the distance between the condenser plates and B is a proportionality factor whose value is to be determined. See Art. 206, page 293,

General Physics. Therefore, using the value of C from equation (4) in the expression $Q = CE$, we get

$$Q = EB\frac{a}{x} \tag{5}$$

But E/x is the intensity of the electric field in the air space between the condenser plates, and $a \times E/x$ is the electric flux Ψ from plate to plate. Therefore

$$Q = B\Psi \tag{6}$$

Differentiating we get

$$\frac{dQ}{dt} = I = B\frac{d\Psi}{dt} \tag{7}$$

Now the magnetomotive force M along any path which encircles I abamperes of current is equal to $4\pi I$*. Therefore, using $M/4\pi$ for I, equation (7) becomes

$$M = 4\pi B \cdot \frac{d\Psi}{dt} \tag{8}$$

The magnetomotive force may be thought of as being taken along the dotted line in the side view in Fig. 36. A changing electric flux through the opening of any loop always means the existence of a magnetomotive force around the loop in accordance with equation (8) which is identical in form to equation (1) except for the presence of the proportionality factor $4\pi B$ and the absence of the negative sign [the positive sign is proper in equation (8) because the magnetic field surrounding an electric current is in the direction in which a *right-handed screw* would have to be turned to travel in the direction of the current]. Considerations almost identical in form to those involved in the discussion of Fig. 184, page 255, *General Physics*, would lead to an equation formally identical in form to equation (3), namely

$$H = 4\pi Bvf \tag{9}$$

where H is the intensity in gausses of the magnetic field pro-

* See pages 79–81, *Advanced Electricity and Magnetism*. Franklin and MacNutt, The Macmillan Co., 1915.

duced by the sidewise motion at velocity v of an electric field of intensity f abvolts per centimeter.

Meaning of opposite signs in equations (3) and (9).—Considering that we have chosen to think of the magnetic field moving to left in Fig. 184, page 255, *General Physics*, as being exactly equivalent to motion of rod bc to the right, and knowing that induced electromotive force is from c towards b when the magnetic field is directed away from the reader, it is evident that the relation between f, v and H as expressed by equation (3) must be as shown in Fig. 37.

Multiplying H by $-v$ in equation (3) turns H in a *clock-wise* direction as seen from the left in Fig. 37.

Multiplying f by $+v$ in equation (9) turns f in a *counter-clock-wise* direction as seen from the left in Fig. 38.

The electromagnetic plane wave.—It is convenient to turn Fig. 38 so that directions of v, H and f are the same as in Fig. 37; thus giving two identical figures, 39 and 40.

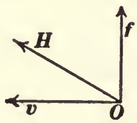

Fig. 37.

H is perpendicular to the plane of the paper and directed away from reader. This figure shows the relation between f, v and H as expressed by equation (3).

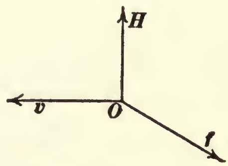

Fig. 38.

f is perpendicular to the plane of the paper and directed towards reader. This figure shows the relation between H, v and f as expressed by equation (9).

Now it is evident that the f which is produced by sidewise motion of H moves along with H because the induced f is always where H is; and, similarly, the H which is produced by sidewise motion of f moves along with f. Therefore:

(*a*) The H in Fig. 40 which is produced by the sidewise motion of f may be the H in Fig. 39, and

(*b*) The f in Fig. 39 which is produced by the sidewise motion of H may be the f in Fig. 40; v being the same in both figures.

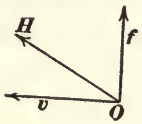

Fig. 39.
f is the electrical field produced by sidewise motion of H at velocity v.

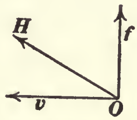

Fig. 40.
H is the magnetic field produced by the sidewise motion of f at velocity v.

Two such mutually sustaining magnetic and electric fields traveling along together constitute a plane electromagnetic wave. A front view (as seen from v) of Figs. 39 and 40 is shown in Fig. 41, in which the full lines represent magnetic lines of force and the dotted lines represent electric lines of force.

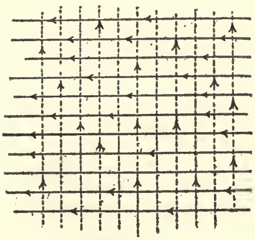

Fig. 41.
Front view of plane elec tromagnetic wave (coming towards reader). Full lines represent magnetic lines of force, dotted lines represent electric lines of force.

It is evident from what is stated above that equations (3) and (9) are to be thought of as simultaneous equations inasmuch as f, H and v have identically the same meanings in both equations. Therefore, multiplying equations (3) and (9) member by member we get

$$v^2 = -\frac{1}{4\pi B} \tag{10}$$

or, ignoring the negative sign which here relates to the quaternion interpretation of the square of a vector quantity, we get

$$v = \sqrt{\frac{1}{4\pi B}} \tag{11}$$

Also, dividing equation (3) by equation (9) member by member and (ignoring negative sign as above) we get

$$\frac{f}{H} = \sqrt{\frac{1}{4\pi B}} \tag{12}$$

Equation (11) gives the velocity at which two mutually sustaining electric and magnetic fields must travel, and equation (12) gives the necessary ratio of the two mutually sustaining fields.

The value of v is 2.99778×10^{10} centimeters per second in air (the velocity of light), and therefore the value of B is 8.84×10^{-23} so that the capacity of an air condenser in abfarads as expressed by equation (4) is

$$C_{\text{in abfarads}} = 8.84 \times 10^{-23} \cdot \frac{a}{x} \tag{13}$$

THE ELECTROSTATIC UNIT OF CHARGE.

According to equation (6) in the above discussion the electric flux which emanates from Q abcoulombs of charge (comes in to the charge if it is negative) is equal to Q/B. This relation is general, and we may apply it to a concentrated positive charge Q'. The electric flux passes out from Q' symmetrically in all directions. Let f be the electric field intensity at a point distant r from Q'. Then $4\pi r^2 f$* is the electric flux from Q' so that

$$\frac{Q'}{B} = 4\pi r^2 f$$

or

$$f = \frac{1}{4\pi B} \cdot \frac{Q'}{r^2} \tag{14}$$

Let another charge Q'' be placed at a point distant r from Q'. Then, according to equation (95), page 308, *General Physics*, fQ'' or $\frac{1}{4\pi B} \cdot \frac{Q'Q''}{r^2}$, is the force F in dynes exerted on Q'' by Q'. That is

$$F = \frac{1}{4\pi B} \cdot \frac{Q'Q''}{r^2} \tag{15}$$

* Argument similar to Art. 135 page 135 *General Physics*.

Consider two equal concentrated charges q (expressed in abcoulombs) *which exert a force of one dyne on each other when they are at a distance of one centimeter apart.* Putting $F = 1$ and $r = 1$ in equation (15), and writing qq or q^2 for $Q'Q''$, we have

$$1 = \frac{1}{4\pi B} q^2$$

or

$$q = \sqrt{4\pi B} \tag{16}$$

But $\sqrt{4\pi B} = 1/v$ according to equation (11) so that equation (16) becomes

$$q = \frac{1}{v} \text{ abcoulombs} \tag{17}$$

The charge q as here specified is the electrostatic unit of charge, and therefore there are $1/v$ abcoulombs in one electrostatic unit of charge.

POTENTIAL.

The velocity potential of a fluid at a point (when it exists) is the height at that point of an imagined hill whose slope is everywhere equal to the fluid velocity. For example, consider a layer of fluid lying on a plane and moving at constant uniform velocity v in the direction of the x-axis of reference. Then if ψ is the height of the potential hill at a point whose abscissa is x we have $\frac{d\psi}{dx} = v$, whence $\psi = vx + a\ constant$.

There are distributions of fluid velocity which have no velocity potential; for example, a layer of water on a rotating disk.

The potential of an electric field at a point (when it exists) is the height at that point of an imagined hill whose slope is everywhere equal to the electric field intensity. For example, the electric potential is $\psi = fx + a\ constant$ in a uniform electric field of intensity f, the x-axis being chosen in the direction of the field. If f is expressed in volts per centimeter and x in centimeters, the height of the potential hill is expressed in volts.

The idea of potential is useful because the precise mode of distribution of an electric field (or of a magnetic field or fluid velocity) in space may be easily thought of and easily formulated in terms of the potential distribution, and especially because certain mathematical transformations, such, for example, as are involved in a change of axes of reference,* are easily made when the field distribution is expressed in terms of potential.

We carefully avoid the use of the word potential in our *General Physics* for three reasons, namely, (*a*) Because electric field (or magnetic field or fluid velocity) is the physically real thing, and potential is merely a mathematical idea, (*b*) Because the

* A good example of such a transformation is given on page 246 of Franklin, MacNutt and Charles's *Calculus*.

idea of potential is useful only when one is considering the space distribution of a field, and such matters are largely beyond the scope of our *General Physics*, and (*c*) Because the common use of the term potential is hopelessly vague and shot through and through with absurdity from the rigorous mathematical point of view. We believe in physical reality, and we believe in mathematical thinking; therefore we cannot tolerate the common use of the word potential.

Let no one imagine that by common usage we refer to the habit of the man in the street. Recently, in discussing the question of potential with a teacher of physics in a technical school, he said "Potential, potential; *you* mean the potential function." Exactly so; but his implication was that *he* always meant something very much simpler than the 'potential function' when he used the term! At a given time temperature has a definite value at every point on the map, and even a complex minded mathematician would be shocked if his inquiry as to the temperature at Cape Cod were to be met by the response "Temperature, temperature; *you* mean the temperature function," and he might reply, somewhat impatiently, "Yes, in the name of all that is precise and definite, that is what I do mean."

It is very greatly to be desired that our teachers of physics and electrical engineering should understand the essentials of what is nowadays called vector analysis, the mathematics of the distribution of scalar and vector values in space. A very simple discussion of this general subject is given on pages 210–253 of Franklin, MacNutt and Charles' *Calculus*.* The idea of electric potential and its use in the study of the electric field is discussed

* This book is recommended by a large western university for use by its students in correspondence courses for private study, it has been used, and to some extent successfully, in the night school and in the high school and in the college, and it has been highly praised as a good book for the graduate student. To such a state of topsy-turvydom has our vague and unintelligible collegiate instruction in the mathematical sciences brought us! Any mathematical treatise that is intelligibly straightforward and definitely clear and precise is, it would seem, good medicine in our time for anybody.

at length on pages 165–192 of Franklin and MacNutt's *Advanced Electricity and Magnetism*, The Macmillan Co., 1915.

71. The electromagnet.—An experiment which is very striking is to exhibit a large and powerful electromagnet. To eliminate the spectacular features, however, let the magnet be of moderate size, consisting of a straight bar of iron or mild steel with a coil of wire upon it. Show that the bar loses nearly all of its magnetism when the exciting current is reduced to zero by opening the supply switch. Magnetize a bar of hardened steel by placing it in the coil and show that it retains a large part of its magnetism when removed from the coil.

72. The heating effect of the electric current, although already familiar to every student, may be strikingly shown by stretching an iron or German silver wire across the lecture table and connecting it through a supply switch to 110-volt supply mains.

73. The chemical effect of the electric current may be shown by placing two lead electrodes in a jar of dilute sulphuric acid and connecting them through a heavy-current rheostat to direct-current supply mains, or a thin coating of copper may be deposited on a bright piece of platinum or silver foil using a solution of copper sulphate as the electrolyte and using a current of not more than, say, half an ampere; an ordinary glow lamp in circuit will provide for a suitable current from 110-volt direct-current mains. See Experiment 84.

74. The magnetic compass.—The most convenient form of demonstration compass is a magnetized steel needle six or eight inches long mounted on a pivot stand.

75. Poles of a magnet.—Dip a bar magnet into a box of iron filings and point out that filings cling only to end parts of bar as shown in Fig. 114, page 187, *General Physics*.

76. Attraction of unlike magnet poles. Repulsion of like magnet poles.—Most young men are familiar with magnetic

attraction, whereas but few are familiar with magnetic repulsion. Mark north-pointing ends of two compass needles by painting them red. Remove one of the needles from its pivot and show that its red end repels the red end of the other needle and attracts the unpainted end of the other needle.

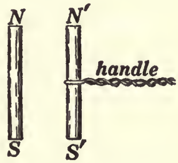

Fig. 42.

Place a short straight cylindrical steel mag net NS, Fig. 42, on a horizontal glass plate in the lantern (vertical projection arrangement) and bring another magnet $N'S'$ near to it, as shown. In one position NS is repelled by $N'S'$, and in the reverse position NS is attracted.

77. Distributed poles and concentrated poles.—Magnetize a short thick bar and a long slim bar of hardened steel, as explained in experiment 71. Dip both magnets into a box of iron filings and point out that filings cling to extended portions near the ends of the short thick bar, but only to short portions near the ends of the long slim bar.

Magnetize a long slim steel bar, remove it from the magnetizing coil, turn it end for end, and bring one end of the bar thus reversed into the end of the coil. Dip the bar into filings and a widely distributed pole will be shown to be spread over the middle portions of the bar, with two moderately concentrated poles at its ends. Show by using a compass needle that the end poles are alike in kind and that the middle pole is unlike the end poles.

Remark.—It may be explained to the student that the idea of the concentrated magnet pole is a *differential*. Thus the element of force action between any two magnet poles is $\Delta F = \frac{dm' \cdot dm''}{r^2}$ so that $F = \int\int \frac{dm' \cdot dm''}{r^2}$, and this equation is a sextuple integral although set in the form of a double integral. Of course it does no good to explain the concentrated pole to the student as a differential, and our easy reference to double and sextuple integrals is made to deter the too eager teacher from making the attempt.

78. The unit pole.—It is a great help to the understanding of Art. 127, page 188, *General Physics*, to have a number of pairs of long slim magnets; taking one pair in the hands at a time, hold them so that two of their poles are about one centimeter apart and the other two poles very far apart (assumed to be indefinitely far apart), pretend to estimate the force of attraction or repulsion. The student will thus see that it is possible to find a pair of magnet poles which attract or repel each other with a force of one dyne when they are one centimeter apart.

79. Magnetic figures.—Place a short bar magnet on a horizontal glass plate in the lantern (vertical projection arrangement), place a thin glass plate over the magnet with steadying wedges or blocks under its edges, dust fine iron filings very sparingly on the thin glass plate and tap the thin glass plate lightly with a hard object like a short glass tube. It is desirable to show also the filaments of filings between two opposite poles of two broad ended magnets as indicated in Fig. 119, page 191, *General Physics*.

80. Behavior of a magnet in a non-uniform magnetic field.—The two poles of a magnet which is placed in a uniform magnetic field (a field which has everywhere the same intensity and of which the lines of force are parallel) are acted upon by equal and opposite forces, as indicated by the arrows in Fig. 120, page 191, *General Physics*. Therefore a uniform magnetic field tends only to turn a magnet into a certain direction, the direction of the field. The two poles of a magnet which is placed in a non-uniform magnetic field (a field which does not have everywhere the same intensity and of which the lines of force are not parallel) are not equal and

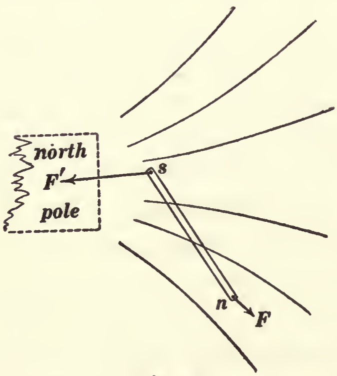

Fig. 43.

opposite, and they tend not only to turn the magnet into the direction of the field but also to impart to the magnet a motion of translation. Thus the two arrows F and F' show the two forces exerted on the small magnet *ns* in Fig. 43.

The *attraction* of a particle of iron by a magnet depends in the first place on the conversion of the particle into a magnet, and in the second place on the non-uniformity of the magnetic field in which the particle finds itself.

The magnetic field near the flat end of a large magnet pole is nearly uniform as indicated by the lines of force in Fig. 44;

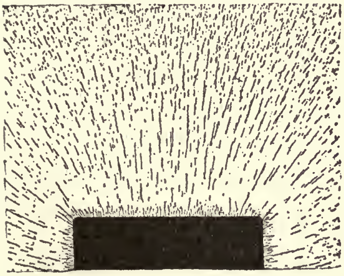

Fig. 44.

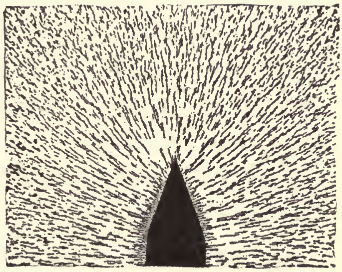

Fig. 45.

near the sharp corners, however, the field is distinctly non-uniform. Therefore small particles of iron are not perceptibly attracted by the flat face of the pole, but only by the sharp corners. Pass the square end of a magnet over a table on which fine iron filings are dusted very sparingly, and the filings are gathered only on the sharp corners of the pole. The lines of force in the neighborhood of a sharply pointed pole diverge strongly as indicated in Fig. 45, that is to say, the magnetic field in the neighborhood of the point is non-uniform to a high degree, and a pointed pole exerts a strong attraction for a small particle of magnetic material. The eye-magnet which is used for removing particles of steel from the eye has a sharp point, and the essential feature of the magnetic ore separator as used to separate the particles of magnetic material from finely crushed ore is a sharply pointed magnet.

81. Oersted's experiment.—Connect a flexible wire (one strand of a lamp cord) to 110-volt direct-current mains through a rheostat so as to have a current of about 2 or 3 amperes in the wire. Stretching a portion of the wire in a north–south direction between the hands, bring it down over a compass needle and note the deflection of the needle. Interpret this experiment as explained in Art. 137, page 197, *General Physics*. Connect a copper plating arrangement in the circuit so as to deposit copper on a bright platinum or silver surface, and point out the fact that, according to the indication of the magnetic needle, the current flows through the copper sulphate solution towards the platinum or silver electrode on which copper is deposited.

Wrap the flexible wire several times around a small soft iron bar, test the magnetic polarity of one end of the bar, and, having previously established the direction of flow of current through wire as above explained, point out the meaning of Fig. 130, page 198, *General Physics*.

82. Side push of the magnetic field on an electric wire.—(*a*) Stretch a 6 or 7-foot span fine copper wire loosely in front of a strong electromagnet as indicated in Fig. 46. The wire should

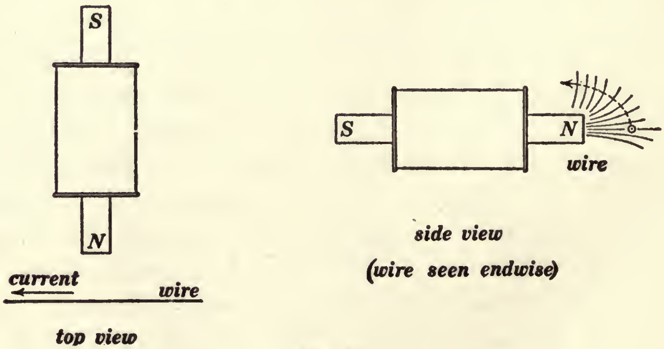

Fig. 46.

be covered with white cotton or silk so as to be easily visible. Connect the wire through a suitable rheostat and through a reversing switch to 110-volt, direct-current supply mains, and

show that the wire is pushed sidewise as indicated by the dotted arrow in the side view. Reverse the current and show that side force is reversed.

It is remarkable how widely prevalent is the idea of *attraction;* point out explicitly that the wire is not attracted by the pole of the magnet but pushed sidewise as indicated.

(*b*) Project the image of a direct-current arc (using ordinary arc lamp carbons) on the screen. Bring the north pole of a bar magnet up behind the arc and call attention to sidewise movement of image of arc on screen. Reverse the magnet and call attention to reversed sidewise movement of image. Bring magnet nearer and nearer to arc until arc is blown out.

(*c*) Arrange a single-pole single-throw switch in a circuit in which 10 or 15 amperes of current flows from 110-volt, direct-current supply mains. Open the switch slowly and call attention to the destructive arc which is produced. Show how greatly this arc is reduced by opening the switch quickly. Place a magnet as indicated in Fig. 140, page 203, *General Physics*, and show that arc is almost instantly blown out if the switch is opened slowly or rapidly.

83. The electric motor.—Exhibit a commercial type of direct-current motor, and take out the armature so that the armature wires may be seen lying on the armature surface parallel to the armature shaft.

The small, non-enclosed, two-pole, ring-wound, Crocker-Wheeler motor is one of the best motors for this experiment. Excite the field magnet of the motor by connecting the field winding through a controlling switch to 110-volt, direct-current supply mains; connect the brushes through a suitable rheostat and a controlling switch to 110-volt direct-current mains; and operate the motor. Show that direction of running is reversed by reversing field connections, or armature connections; but that reversal of both leaves direction of running unchanged.

If a motor having a drum-wound armature is used, the student

9

should be required to read the fine print on page 209, *General Physics*.

84. The chemical effect of the electric current may be strikingly shown by the decomposition of lead nitrate as explained in Art. 147, page 214, *General Physics*. The best arrangement is to use a narrow cell with plate glass sides and place it in the lantern (horizontal projection arrangement). After the deposition of lead crystals on one electrode, reverse the battery connections when these crystals will be seen to redissolve, and crystals will grow on the other electrode.

Where lead crystals are being deposited the density of the solution is reduced as may be seen by the upward streaming of the solution near the cathode, and where the previously deposited crystals are being redissolved the density of the solution is increased as may be seen by the downward streaming of the solution near the anode. Point out that the chemical action in an electrolytic cell takes place only at the electrodes.

The voltaic cell.—It is very important to direct the students' attention very particularly to Art. 149, page 216, *General Physics*. *A voltaic cell is any electrolytic cell in which the chemical action produced by the current is a source of energy.*

85. The simple voltaic cell. Voltaic action and local action.—Place the above described glass cell in the lantern (horizontal projection arrangement), fill the cell with dilute sulphuric acid, and introduce a strip of clean pure zinc. No perceptible chemical action takes place, that is to say, no bubbles of hydrogen are seen to be produced.

Connect the strip of zinc to a strip of copper by a soldered wire connection and place the two strips into the cell. An electric current now flows through the wire from copper to zinc and through the acid from zinc to copper, this current decomposes the H_2SO_4, the SO_4 radical is liberated at the zinc electrode where it combines with the zinc to form $ZnSO_4$, and the hydrogen is liberated at the copper strip where it appears in the form of

bubbles. Cut the wire and this action ceases. This is voltaic action.

Dip the clean strip of zinc momentarily into a dilute solution of copper sulphate, and minute particles of copper will be left on the zinc. Replace the contaminated strip of zinc in the dilute sulphuric acid, and chemical action takes place as indicated by the formation of hydrogen bubbles. This is local action.

Connect the strip of contaminated zinc to a strip of copper by a soldered wire connection and place both strips in the sulphuric acid. Hydrogen bubbles will be seen to form at both electrodes, that is to say, local action and voltaic action both take place.

Amalgamate the contaminated strip of zinc by rubbing a little mercury over it, replace the two strips in the sulphuric acid cell, and voltaic action, only, will be in evidence by the liberation of hydrogen bubbles at the copper strip. Cut the wire and this voltaic action will cease.

86. Experiment showing the use of oxidizing agent around the cathode (copper or carbon) of a voltaic cell.—This matter is explained in Art. 153, page 220, *General Physics*, and it may be shown experimentally as follows: Connect an ammeter to a simple voltaic cell, and allow the current to flow until the ammeter reading falls considerably (until the dissolved atmospheric oxygen near the copper electrode is used up and hydrogen bubbles are given off copiously). Then pour into the cell a solution of potassium bichromate. The ammeter reading is thereby increased and hydrogen bubbles are no longer liberated.

87. Change of resistance with temperature.—(*a*) A battery jar is provided with two lead electrodes. Fill the jar nearly full of water, connect through an ammeter (reading up to 20 or 30 amperes) to 110-volt direct current supply mains, and connect a voltmeter between the electrodes (110-volt meter). Place a thermometer in the jar, and pour dilute sulphuric acid into the jar *very slowly* with vigorous stirring until the ammeter reading

is about 5 amperes. Then read thermometer, voltmeter and ammeter.

Allow the current to flow for a while until the temperature rises considerably. Then again take the readings of thermometer, voltmeter and ammeter.

Ignoring the small polarization voltage (2 or 3 volts) the resistance of the acid solution at each temperature can be found from the corresponding readings of voltmeter and ammeter.

(*b*) File off the corners of three $4'' \times 8''$ pieces of window glass. Wind about 100 feet of No. 28 B. & S. cotton-covered copper wire in one layer on each strip, leaving 3 inches of one end of each piece of glass uncovered. Clamp the bare ends of the glass strips to a wooden block, connect all of the windings of wire in series and bring the terminals to two binding posts on the wooden block. Place the wooden block across the top of a battery jar with the wire-covered strips of glass projecting into the jar. Fill the jar with kerosene so as to cover the wire windings, bore a hole through the wooden cover for a thermometer, and arrange a good stirrer.

Connect as under *a* above to 110-volt direct-current supply mains, and determine the resistance of the copper wire at, say, 25° C. and at 75° C.

88. Conductivity of glass at high temperatures.—A thin-walled glass tube about one inch in diameter and 8 or 10 inches long is connected as shown in Fig. 47, and one side of the tube is

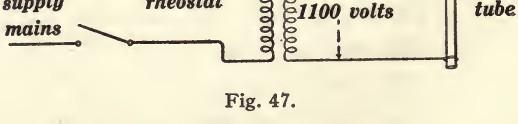

Fig. 47.

heated by sweeping the flame of a blast lamp up and down between the wire terminals. Keep the supply switch open until all connections are made, and use great caution to avoid contact

with high-voltage circuit after the supply switch is closed. The current starts along a very narrow path, quickly heats the narrow strip to a very high temperature, and the resistance of this path of very hot glass drops to 200 ohms or less. The 10-ohm, 10-ampere rheostat is necessary to prevent an excessive flow of current after the glass begins to conduct appreciably.

Electricity or energy; which?—Article 170, page 238, *General Physics*, is intended to turn the student away from any premature incursion into the atomic theory of electricity, and hold his attention fast to the most important practical phase of electrical science, namely, the purely mechanical phase. It is conceivable that the atomic conceptions of electrical phenomena may some time come to be important in everyday life and in everyday engineering, but that time is certainly not yet; although the atomic theory is nearly as important in engineering research as in any other branch of physical research.

89. Experiment with fan blower and dynamo generator.—

Drive a fan blower by a small, direct-current, shunt motor with an ammeter in the armature circuit of the motor to indicate roughly the power delivered to the blower.	Drive a small shunt dynamo as a generator by means of a small, direct-current, shunt motor with an ammeter in the armature circuit of the motor to indicate roughly the power required to drive the generator.
Arrange a gate so that the outlet of the blower can be closed or opened to any desired extent.	Arrange a rheostat and switch so that the generator can be operated on open circuit (armature delivering no current) or so that the current delivery can be varied at will.
Arrange an open-tube manometer to measure the air pressure behind the gate.	Connect a voltmeter across the armature terminals of the generator.

With gate closed note power required to drive the fan and note value of air pressure E.

With armature circuit of generator open note power required to drive the generator and note voltage E across terminals of generator armature.

Opening gate wider and wider, note increase of power required to drive the fan and note slight decrease of pressure E behind gate.

Closing switch and adjusting rheostat so as to take more and more current from generator armature, note increase of power required to drive generator and note slight decrease of voltage E between armature terminals.

An interesting calculation is that which is suggested in problem 198, page 568, *General Physics*, and approximate data for such a calculation can be obtained from the motor-generator arrangement here described if an ammeter is placed in the armature circuit of the generator. Thus if 5 amperes of current is taken by the motor armature from 110-volt direct-current supply mains when generator is delivering no current, and if 15 amperes is taken by motor armature when the generator is delivering 9 amperes. Then, neglecting RI^2 losses in motor armature and in dynamo armature, we may take (15 amperes − 5 amperes) × 110 volts as the part of the power which is supplied to the generator by the motor and delivered as output by the generator so that the voltage E of the generator must be approximately such as to give

$$9 \text{ amperes} \times E = (15 \text{ amperes} - 5 \text{ amperes}) \times 110 \text{ volts}.$$

90. Experiment on back electromotive force of a motor.—Connect the field winding F of a direct-current shunt motor directly to 110-volt direct-current supply mains through a controlling switch S; connect the armature M of the motor to the

same mains through a rheostat, an ammeter A and a controlling switch S'; and connect a voltmeter V to terminals of M as shown in Fig. 48.

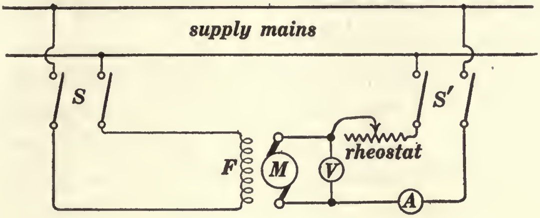

Fig. 48.

To determine resistance of M lock the armature so that it cannot rotate, and reduce resistance in the rheostat until the rated full-load current flows through A and M; then take readings of A and V. The required resistance of M is equal to V/A according to Ohm's law.

Then unlock the motor armature and allow it to run, without a belt load, let us say, and note the great decrease of current through A and M and the great increase of voltage indicated by V. Take readings A' and V' when motor is running at full speed.

We thus get data for a calculation like that of problem 215, page 571, *General Physics*.

Note.—The important thing in this experiment is to show the great decrease of current through M when the motor is allowed to start and rise to full speed.

91. The transformer experiment.—A laminated iron core about $1'' \times 1'' \times 8''$ long has about 150 turns of No. 16, B. & S. copper wire wound upon it, the wound core is mounted in an upright position on a wooden base, and the terminals of the winding are brought out to binding posts as indicated in Fig. 198, page 266, *General Physics*. This winding can be safely connected directly to 110-volt, 60-cycle, alternating-current supply mains.

(*a*) A copper ring 2 or 3 inches in diameter, $\frac{1}{4}$ inch wide and $\frac{1}{8}$ inch thick is held in a pair of strong pliers and brought down over the above wire-wound core. The repeated reversals of magnetism of the core induce an alternating current in the ring and the ring becomes red hot.

(*b*) Hold the cold ring loosely by the fingers so as to encircle the core about 2 inches from its upper end, and suddenly close the supply switch. The ring is thrown violently upwards.

(*c*) A small coil of insulated copper wire three or four inches in diameter and containing 15 or 20 turns of wire has a short piece of German silver wire (No. 22, B. & S., let us say) connected across its terminals, and the coil is held in the hand and brought slowly down over the above wire-wound core. The small coil acts as the secondary coil of a transformer (a step-down transformer), and an alternating current is induced in it as indicated by the heating of the German silver wire.

92. Eddy currents in a solid iron rod.—A coil of insulated, No. 16, B. & S., copper wire containing about 1,000 turns and having an opening about 3 inches in diameter is connected to 110-volt, 60-cycle, alternating current-supply mains through a suitable rheostat (seven or eight ohms and ten-ampere capacity) and a controlling rheostat. A short iron rod 2 or $2\frac{1}{2}$ inches in diameter with a strong handle like a soldering iron is thrust into the opening of the coil, and the rod soon becomes very hot because of eddy currents.

Eddy currents due to motion of a good electrical conductor near an unvarying magnet may be shown by dropping a thick sheet of copper into the space between the two facing poles of a strong electromagnet.

93. Demagnetization by a weakening series of reversals.—A steel magnet placed in the coil which is described in experiment 92 and slowly withdrawn is left almost wholly demagnetized. A magnet (or a magnetized watch) may be demagnetized by holding it near the pole of a strong magnet and turning it slowly

while moving it farther and farther away from the magnet pole. The axis of turning must be at each place at right angles to the lines of force of the magnetic field at that place.

94. Spark at break.—Connect the coil and core which are described in Experiment 96 to two cells of storage battery in series, and arrange a strip of spring brass *BB* as shown in Fig. 49 for quickly breaking the circuit at *p* (see also Fig. 215, page 276, *General Physics*). Also arrange a piece of German silver wire of the same resistance as the coil to be put in circuit in place of the coil and permit the same steady current to flow. A small glow lamp may be included in the circuit to show by its brightness that the current has approximately the same value in each part *a*, *b* and *c* of the experiment, as follows:

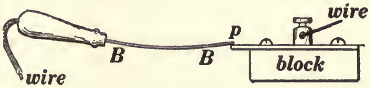

Fig. 49.

(*a*) With German silver wire in place of the coil, break the circuit suddenly at *p* and note that there is no perceptible spark at break.

(*b*) Place the coil in circuit without the laminated iron core, and show that there is a moderate spark at break

(*c*) Place the laminated core in the coil and show that there is a very intense spark at break.

An interesting experiment for the individual student is to connect a telegraph relay as a buzzer (see problem *General Physics*), operate it by three or four dry cells, and connect hand-electrodes to the terminals of the magnet winding. At each break of the circuit a moderately high voltage exists across the magnet winding which may be felt by taking hold of the hand electrodes.

95. Choking effect of an inductance in an alternating-current circuit.—Connect a 60-watt, 110-volt tungsten lamp to 110-volt, 60-cycle, alternating-current supply mains in series with the German silver resistance referred to in the previous experiment.

Connect a similar lamp to the same supply mains in series with the coil referred to in the previous experiment. The resistances of the two circuits are the same, but the lamp which is in series with the coil is dimmer than the other lamp (showing that less alternating current flows through it), and if the laminated iron core is placed in the coil the current will be very greatly reduced. See page 279, *General Physics*.

Note.—Change-over switches may be arranged to shift the above circuits from 110-volt, alternating-current supply mains to 110-volt, direct-current supply mains.

96. Experiment showing slow growth of current in an inductive circuit.—A small glow lamp is connected to a battery through a controlling switch and in series with a coil of wire in which is a removable iron core. When the switch is closed the lamp comes almost instantly to full brightness when the iron core is removed, but it takes a very perceptible time for the lamp to come to full brightness after closing the switch when the iron core is in place.

A delay of about 0.6 second with iron core in place will be obtained under following specifications: Use a half-ampere, 2-volt, tungsten filament lamp, and two storage battery cells connected in series. Make the coil about 6 inches long with 1 inch depth of winding and containing about 1,350 turns of No. 14, B. & S., double cotton-covered copper wire. Make the core $1\frac{1}{2}$ inch $\times$ $1\frac{1}{2}$ inch $\times$ 13 inches long, built up of soft sheet-iron stampings bound together by one layer of strong twine. The corners of the core should be rounded off to avoid cutting of twine. Soak the whole core in shellac varnish and bake. Each end of the core for about $3\frac{1}{2}$ inches should be bare so that the edges of the laminations of the core can come as close as possible to the edges of laminations of a yoke. Make area of contact of each end of core with yoke about $1\frac{1}{2} \times 3$ inches. The coil must have a $2\frac{1}{2}$-inch hole to admit the laminated core. The yoke should be mounted on a base board with lugs projecting upwards to connect with and support the ends of the core.

A very interesting experiment with the above apparatus is the following: Connect lamp and coil in series to the 2 cells of storage battery, placing the coil on its side on the table (yoke not used). Allow the magnetic forces to draw the core quickly into the coil and the lamp will be momentarily dimmed. Draw the core quickly out of the coil and the lamp will become momentarily very brilliant. (Be careful not to burn out the lamp by withdrawing core too quickly.)

While the core is being drawn into the coil the power, EI, delivered to the coil by the battery is being only in part used to heat the wire of the coil (RI^2) because some power is evidently used to pull the core into the coil. Therefore RI^2 is less than EI, or RI is less than E, or I is less than E/R, which is the value of the steady current according to Ohm's law. While the core is being drawn into the coil the magnetic flux through the coil is increasing, this increasing flux induces a back electromotive force in the coil (opposing the current), and the work done in forcing the current to flow in opposition to this back electromotive force is spent in two ways, namely, in pulling the core into the coil and in magnetizing the core.

Inductance of a coil or circuit.—The discussion of inductance as given in *General Physics* is seriously incomplete. Indeed we have never written a book without being unduly influenced by a feeling of the hopelessness of the undertaking of bringing young men to a fairly complete understanding of the fundamentals of physics, and we therefore cut out things that should be included. We might excuse such omissions by saying that we do not wish to make a perfect book and thereby deprive all physics teachers of their work!

A coil, having Z turns of wire, has a current i flowing in it, as indicated in Fig. 50. The current in the coil produces a magnetic field in the surrounding region as indicated by the lines of force in Fig. 50, and it can be shown* from the magnetic definition of current strength (see Art. 144, page 209, *General*

* It is not worth while to introduce this proof here.

Physics) that the intensity of the magnetic field is everywhere doubled in value if i is doubled in value, the trend of the lines of force remaining unchanged. Therefore the amount of magnetic flux through each turn of wire or *the average flux per turn*, Φ, is doubled if i is doubled. Therefore Φ is proportional to i so that we may write

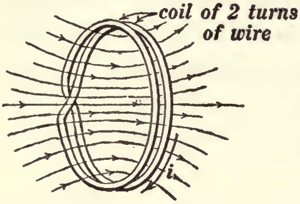

Fig. 50.

$$\Phi = bi \tag{i}$$

where b is a constant for a given coil or circuit. If i changes it is evident that Φ must change b times as fast, or

$$\frac{d\Phi}{dt} = b\frac{di}{dt}. \tag{ii}$$

But the changing Φ induces an electromotive force $-\frac{d\Phi}{dt}$ in each turn of wire or a total electromotive force $-Z\frac{d\Phi}{dt}$ in the whole coil, according to equation (82), page 257, *General Physics*, or using the value of $\frac{d\Phi}{dt}$ from (ii) we get $-bZ\frac{di}{dt}$ as an expression for the electromotive force induced in the coil by the changing current, and, if we represent the quantity bZ by the single letter L, we have

$$E' = -L\frac{di}{dt} \tag{iii}$$

where E' is the electromotive force induced in a coil or circuit by the changing of the current. The negative sign indicates that E' is *opposed* to i when $\frac{di}{dt}$ is positive, that is, when i is increasing. Therefore, to *make* the current increase at the rate $\frac{di}{dt}$, an outside electromotive force (the electromotive force of a battery, for example) *equal to* $L\frac{di}{dt}$ *but helping the current*

must act or push on the circuit. That is

$$E = L\frac{di}{dt} \tag{iv}$$

which is equation (84), page 278, *General Physics*, and in which E is the outside electromotive force required to make the current in a coil or circuit increase at the rate $\frac{di}{dt}$, and L is a constant for the given circuit. The quantity L is called the *inductance* of the circuit.

Let R be the resistance of the circuit, and let E be the total electromotive force acting on the circuit. Then a portion, Ri, of the electromotive force is used to overcome resistance, and the remainder, $E - Ri$, causes the current to increase in accordance with equation (iv).

Note.—If there is an iron core in the coil in Fig. 50 the flux Φ will be nearly proportional to i for small values of i, but as the iron approaches magnetic saturation the flux Φ is no longer even approximately proportional to i.

Flux-turns.—The quantity Φ in the above discussion is the average flux per turn of wire, and the product ΦZ might properly be called the *total flux* passing through the coil (each part of the actual flux in Fig. 50 being counted n times, where n is the number of turns of wire surrounding that part). This product ΦZ is usually called the *number of linkages of lines of force and turns of wire*, and it is expressed as *flux-turns*. Multiply both members of equation (i) by Z and we get $\Phi Z = bZi$. But bZ is the inductance L of the coil, so that

$$\Phi Z = Li. \tag{v}$$

Consider a canal boat on which a propelling force E is exerted by the tow rope. Assume the backward drag of the water to be proportional to the

Consider a circuit on which an electromotive force E is acting. The portion of E which is used to overcome friction is Ri, when R is a constant and

velocity i of the boat. Then the net accelerating force is $E - Ri$, where R is a constant, and we have

$$E - Ri = L\frac{di}{dt} \qquad \text{(vi)}$$

where L is the mass of the boat and $\frac{di}{dt}$ is its acceleration.

When the boat reaches full speed (corresponding to given force E) then $\frac{di}{dt} = 0$ and equation (vi) becomes

$$E = Ri \quad \text{or} \quad i = \frac{E}{R} \qquad \text{(viii)}$$

At the instant that the mule begins to pull, $i = 0$ and equation (vi) becomes

$$E = L\frac{di}{dt} \qquad \text{(x)}$$

When the mule stops pulling after having set the boat in motion, $E = 0$ and equation (vi) becomes

$$L\frac{di}{dt} = -Ri \qquad \text{(xii)}$$

i is the current, and the net accelerating electromotive force is $E - Ri$, so that

$$E - Ri = L\frac{di}{dt} \qquad \text{(vii)}$$

where L is the inductance of the circuit and $\frac{di}{dt}$ is the rate of increase of the current.

When the current reaches full value (corresponding to given value of E) then $\frac{di}{dt} = 0$ and equation (vii) becomes

$$E = Ri \quad \text{or} \quad i = \frac{E}{R} \qquad \text{(ix)}$$

At the instant that the electromotive force E begins to act on the circuit, $i = 0$ and equation (vii) becomes

$$E = L\frac{di}{dt} \qquad \text{(xi)}$$

When the electromotive force E ceases to act after having established a current, $E = 0$ and equation (vii) becomes

$$L\frac{di}{dt} = -Ri \qquad \text{(xiii)}$$

Note.—Equation (vi) is usually written and thought of in the form $E = Ri + L\frac{di}{dt}$, that is to say, the total elcetromotive force acting on a simple circuit of resistance R and inductance L is partly used to "overcome" resistance and partly used to "overcome" inductance; the former part is Ri and the latter part is $L\frac{di}{dt}$.

97. Elimination of spark at break by a condenser.—Figure 51 shows the coil L and the quick-break arrangement p (as described in experiment 94) connected to 2 storage battery cells

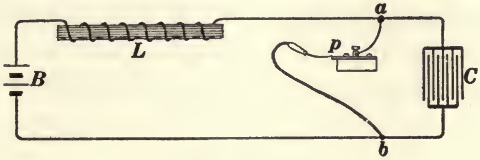

Fig. 51.

B. As stated in experiment 94 an intense spark at break is produced when the iron core is in the coil, but when a condenser C is connected as shown the spark at break is eliminated. The condenser should have about one microfarad capacity.

The reversed surge of current which flows through the coil L and the battery B after breaking the circuit at p may be shown by its demagnetizing action on the iron core as explained on page 283, *General Physics*.

The so-called discharge resistance.—Connect a glow lamp in place of the condenser in Fig. 51 and show that spark at break is eliminated. It is common practice to use a resistance thus connected across a switch which is used to open a highly inductive circuit, such as the field winding of a large dynamo, and the switch is always arranged as a two-step switch. The first step inserts the resistance in circuit and the second step opens the circuit after the current has been greatly reduced by the insertion of the resistance. A common arrangement of switch with "discharge resistance" is shown in Fig. 52.

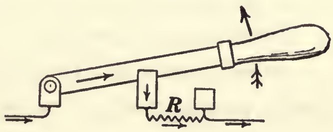

Fig. 52.

98. Flow of alternating current through a circuit containing a condenser.—Connect a 25-watt, 110-volt, tungsten filament, glow lamp in series with a 10- or 20-microfarad condenser, and arrange a change-over switch so that the condenser and lamp

can be connected to 110-volt, direct-current, supply mains or to 60-cycle, 110-volt, alternating-current, supply mains as indicated in Figs. 222 and 223, page 284, *General Physics*.

99. The lightning arrester.—A very interesting experiment is to show a simple type of lightning arrester in action. Thus Fig. 53 shows a single-pole direct-current arrester of the magnetic

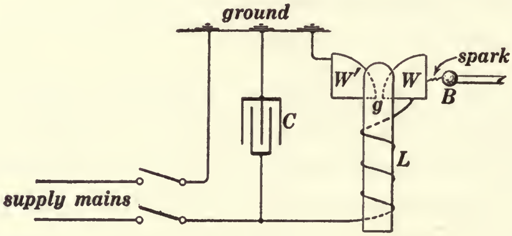

Fig. 53.

blow-out type. The ground connections in the figure should be good metallic connections to the same piece of gas or water pipe because the action of the magnetic blow-out is most interesting when the spark across gap *g* produces a dead short circuit. If 220-volt direct-current supply is available it should be used in preference to 110-volt supply.

Connect one terminal of the electric machine or induction coil to ground and the other terminal to the metal ball *B*. When the spark jumps from *B* to *W* it is choked by the winding *L*, and therefore the sudden spark discharge passes across gap *g* to ground. But the spark across *g* starts an electric arc and introduces a dead short circuit between the supply mains. The blow-out magnet *L* is to eliminate this short circuit.

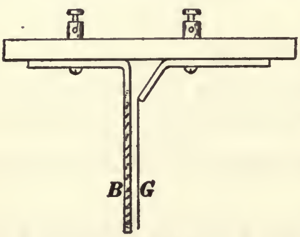

Fig. 54.

100. Electrostatic attraction. — A brass strip *B* (faced with varnished paper so as to avoid short-circuit when *G* touches it), and a suspended strip of

gold leaf G as indicated in Fig. 54 are placed in the lantern (horizontal projection arrangement), and the binding posts are connected to 110-volt, direct-current supply mains.

Show that the attraction is sensibly the same if the binding posts are connected to 110-volt, alternating-current supply mains.

Exhibit an electrostatic voltmeter.

101. Capacity of a condenser.—The experiment which is described in Art. 204, page 289, *General Physics*, is very helpful in giving to the student a clear understanding of equation (89), page 290, *General Physics*.

The term *capacity* is misleading in that it suggests that a condenser can hold only a certain amount of charge just as a pail can hold only a certain amount of water.

102. Residual charge on a condenser. So-called electric absorption.—

A rubber tube is stretched by a very considerable steady force E and certain elongation Q is produced.	A Leyden jar condenser is charged by a very high steady voltage E and certain charge Q is drawn out of one coating and pushed into the other coating thus producing what may be thought of as an electrical stress in the glass wall.
The end of the rubber tube is momentarily released so as to be free to move and the stretch of the tube is relieved by the movement.	The two coatings of the Leyden jar are connected by a conductor so that current can flow, and the charge on the coatings together with the electrical stress in the glass wall disappears.
The free end of the momentarily relieved rubber tube is then made fast so that it cannot move.	The wire connection is then taken away so that current cannot flow.

A minute later the end of the tube is released and the movement of the end shows that a small amount of *stretched condition* has developed in the tube.

A minute later the two coatings are connected by wire and a minute spark shows that a small amount of *charged condition* has developed.

When a stretched rubber band is momentarily released most of the stretch disappears at once, but a small residue of stretch disappears very slowly; and this residue of stretch accumulates as sensible stretch if the momentarily released end of the rubber band is quickly made fast.

When the plates of a charged condenser are momentarily connected by a wire, most of the electrical stress in the dielectric disappears at once, but a small residue of the stress disappears very slowly; and this residue of stress accumulates as sensible charge on the condenser plates if the momentary wire connection is taken away so that no current can flow.

103. The use of the spark gauge.—The simple experiment which is described in Art. 210, pages 298–299, *General Physics*, is best carried out by the student in the laboratory.

104. The ringing spark.—The oscillatory character of the discharge of a condenser through an inductance can be shown by

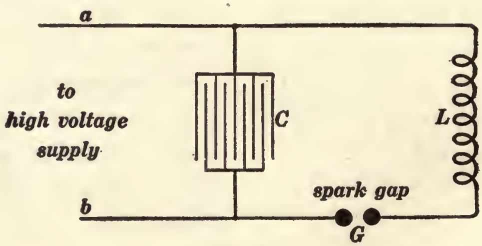

Fig. 55.

the arrangement shown in Fig. 55. The condenser C is charged to about 20,000 or 30,000 volts by a large influence machine, and when it discharges across the spark gap the back and forth

surging of the discharge produces 15 or 20 sparks in rapid succession and if the frequency is low enough this multiple spark gives a very distinct ringing sound like the stroke of a hammer on an anvil. Also the spark when reviewed in a rotating mirror is seen to consist of a series of sparks in rapid succession.

The pitch of the tone emitted by the spark can be easily determined by adjusting a calibrated Galton whistle to the same pitch.

It is interesting to vary the pitch of the spark by using fewer Leyden jars (smaller capacity) or by using a portion only of the sections of the coil (smaller inductance). It is advisable therefore to bring out terminals from intermediate pancake coils of the coil L.

A condenser C, Fig. 55, consisting of 6 fairly large Leyden jars grouped in parallel and a coil L having about 2,500 turns of double cotton covered No. 18, B. & S., copper wire wound on a spool with winding space 8 inches long and 1 inch deep, inside diameter of winding 3 inches, outside diameter of winding 5 inches, will give a frequency of oscillation of about 2,000 cycles per second, and at this frequency the spark will give a tone having a pitch of 4,000 complete vibrations per second which is suitable for this experiment.

Note.—Very high voltages come into existence across the terminals of L in this experiment and therefore the coil should be wound as a set of, say, 20 pancake coils each $3''$ inside diameter $\times$ 5 inches outside diameter $\times$ $\frac{1}{2}$ inch thick, and these pancake coils should be assembled on a hard rubber tube 2 inches inside diameter and about $2\frac{3}{4}$ inches outside diameter with thin disks of wood between; and the whole coil should be impregnated with beeswax and rosin compound.

105. The use of the electric arc for the maintenance of electric oscillations.—An electric arc is maintained between a carbon rod A and a water cooled copper vessel B, and the arc is shunted by a circuit containing an inductance L and a con-

denser C as shown in Fig. 56. Under these conditions a steady back and forth surging of current through the circuit LC is maintained.

Use as an inductance a winding of about 75 turns of No. 16, B. & S., cotton-covered copper wire wound in five or six sections around a short wooden cylinder 6 inches in diameter, and bring out terminals from intermediate sections so as to be able to use 15 or 30 or 45 or 60 or 75 turns at will. Use one one-microfarad condenser or two such condensers connected in parallel.

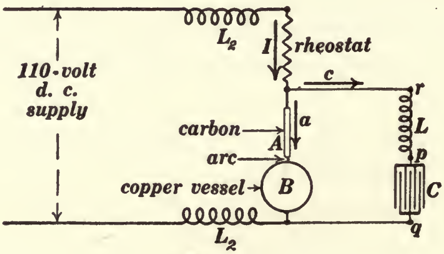

Fig. 56.

With the two condensers and with all of the 75 turns in circuit the frequency of oscillation will be about 4,000 or 5,000 cycles per second and the arc will emit a shrill whistling sound of which the pitch is 8 or 10 thousand complete vibrations per second.

Make an open coil of 10 or 12 turns of No. 18 insulated copper wire, the coil being large enough to slip over the coil L as above described, connect a small glow lamp to the terminals of this coil, and bring the coil slowly over the coil L while the arc is whistling. Be careful not to burn out the lamp. Under these conditions the auxiliary coil acts as the secondary of a transformer as indicated by the lighting of the lamp.

Reduce the value of L by using fewer sections as above described, or reduce C, or both, and the tone emitted by the arc will rise higher and higher in pitch and eventually pass beyond the pitch limit of audibility; but the oscillatory char-

acter of the current through L and C will still be shown by the lamp.*

This experiment operates more satisfactorily if the arc is in an atmosphere of hydrogen, but it usually operates quite satisfactorily with the arc in air. A few trials is sufficient to make it go, especially if the current through the arc is reversed once or twice to get the more favorable direction.

The inductances L_2 L_2 are helpful in that they tend to keep the supply current at a constant value.

106. The electric field.—The region around a charged body is an electric field, and a match stick suspended by a fine thread may be used for indicating the direction of electric field in a manner very similar to the use of the compass needle for indicating the direction of a magnetic field. See page 305, *General Physics*.

107. Experiment giving a basis for the definition of intensity of an electric field.—See pages 306–308, *General Physics*.

A very amusing experiment is to place small pith figures between two metal plates AA and BB as shown in Fig. 57, the plates being connected to the terminals of a small influence electric machine. The pith figures should be made slightly conducting by rubbing a trace of salt on them, and the plates AA and BB should be slightly dished as shown.

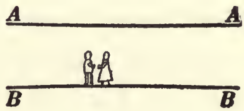

Fig. 57.

108. The rosin experiment is described on page 309, *General Physics*. To get the best results the metal ladle should be connected to one terminal of a large influence machine, and the other terminal of the machine should be grounded.

The gold-leaf electroscope.—A very convenient arrangement of the gold-leaf electroscope is shown in Fig. 58. The leaves l

* The explanation of the production of oscillations by the arc in Fig. 56 is given on pages 141–142 of W. S. Franklin's *Elements of Electrical Engineering*, Vol. I, The Macmillan Co., 1917.

of the electroscope are in the field of the lantern, and the metal plate P of the electroscope is connected by a fine wire to an insulated metal plate P' which stands on the lecture table. The fine wire should be held loosely by small weights as shown, and the insulating supports S and S' should be blocks of cast

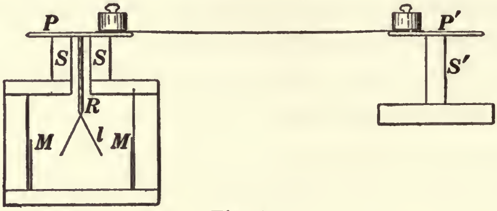

Fig. 58.

sulphur and their surfaces should be freshly scraped. The support S should be made of four brick-like slabs (as shown in section) thus shielding the gold leaves from currents of air that might enter the case around the rod R. MM are metal strips connected to earth. Of course the case has glass front and back.

In using a gold leaf electroscope it is usually most convenient to charge the electroscope by influence, using a glass rod which is positively charged by rubbing it with silk, or a hard-rubber rod which is negatively charged by rubbing it with fur.

109. Charging by influence.—Bring a positively charged glass rod near plate P', Fig. 58, touch P' with the finger and withdraw the charged glass rod, and the entire system PP' will be left negatively charged.

110. Use of the gold-leaf electroscope.—(*a*) When the system PP', Fig. 58, is neutral a positive or a negative charge brought near to P' will cause the gold leaves to spread apart.

(*b*) When the system PP' is charged, a *like* charge brought near to P' increases the spread of the leaves.

(*c*) When the system PP' is charged, an *unlike* charge, if large enough, brought nearer and nearer to P' lessens the spread of the leaves to zero and then causes the leaves to spread apart again as it comes still nearer to P'.

Note.—These effects should be explained, let us say, in terms of the two fluid theory so as to enable the student to hold them in mind for a while at least. The behavior of the gold-leaf electroscope is complicated by changes of capacity as may be understood from experiment 114, and therefore the statements under (*b*) and (*c*) should in all strictness be qualified.

111. Sharing of charge by two insulated metal balls brought into contact.—Take two similar metal balls 4 or 5 inches in diameter with insulating handles. Charge one ball positively, let us say, note its effect on gold leaves when it is brought near to P' in Fig. 58 (electroscope being already slightly charged), touch it to the other ball, and then show that the two balls have nearly equal positive charges, and that the charge on each ball is considerably less than what was initially on the first ball.

Note.—The greatest amount of charge that can be held on a 5-inch metal ball in air would produce about a thirty-thousandth of an ampere for a thousandth of a second and therefore one cannot easily study the transfer of charge from one ball to the other by measuring the electric current which is associated with (which indeed constitutes) the transfer of charge.

112. Giving up of entire charge by internal contact.—Charge a metal ball and touch it to the inside of a metal can which is itself supported by an insulating handle, and show that the ball is left almost entirely without charge whereas the charge on the can increases more and more with each internal contact of the freshly charged ball.

113. The electric doubler.—The experiment which is described in Art. 221, page 315, *General Physics*, is very interesting and instructive. Strands of loose cotton threads serve very well in place of the slabs of pith. The threads should be drawn repeatedly between the fingers to make them slightly conducting.

It is instructive to place the metal balls in the position shown in Fig. 254, page 315, *General Physics*, and show that they have approximately equal but opposite charges when they are sepa-

rated and withdrawn. This experiment shows charging by influence in its most complete and simplest form.

114. Change of capacity of plate P' in Fig. 58 and its effect on the gold leaves. Demonstration of inductivity.—

Two elastic bags A and A' are connected by a pipe, and air is pumped into the bags so as to inflate them.

Fig. 59.

If the walls of bag A' could be weakened by making them thinner some of the air would flow out of A into A', thus reducing the amount of air in A.

If the material of bag A' could be made of more yielding stuff (without changing its thickness) some of the air would flow from A into A'.

Two insulated metal bodies P and P' are connected by a wire and charged with electricity.

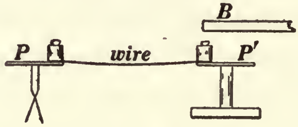

Fig. 60.

If the dielectric which surrounds P' is made more yielding by making it thinner some of the charge will flow out of P into P', thus reducing the amount of charge on P and decreasing the spread of the gold leaves. By bringing a grounded metal body near to P' as shown the specified effect is produced.

If a slab of paraffine is placed between P' and B the air dielectric is replaced to some extent by a more yielding dielectric (a dielectric with greater inductivity) and some of the charge will flow out of P into P', thus reducing the spread of the gold leaves.

115. The corona discharge may be shown in a dark room by stretching two very fine wires two to six inches apart and connecting them to an influence electric machine or to the secondary coil of a step-up transformer giving a secondary voltage of twenty or thirty thousand volts.

The familiar old-time name of the corona discharge is *brush discharge*, and it is seen on the projecting points on an influence machine which is operated in a dark room.

116. Electrical smoke deposition.—This experiment is described and explained in Art. 225, page 319, *General Physics*.

117. The ozonizer is described and its action is explained in Art. 226, page 320, *General Physics*.

Discharge of electricity through gases and radio-activity.—One of the best teachers of elementary physics in the United States, who is well known for the splendid research he has done in recent years, once remarked to the authors that he never could find the time to devote to the discharge of electricity through gases and to the atomic theory of electricity in his rather severe course in elementary physics (the authors had admitted their own difficulties in this respect); but, he added, "I often have a spare lecture hour or two near the end of the term which enables me to show and discuss a few experiments, and," he added, in a highly significant semi-humorous vein, 'I generally prepare myself for these lectures by looking up the subject in ————'s High School Physics."

Following is a list of experiments which we have found satisfactory:

(*a*) Show the decreasing dielectric strength of a gas (air) with decreasing pressure by connecting a vacuum tube in parallel with an adjustable spark gap, and exhausting the vacuum tube slowly.

(*b*) Show the characteristic Geissler tube discharge.

(*c*) Show the characteristic Crookes tube discharge.

(*d*) Exhibit cathode ray shadow and show magnetic deflection

of cathode rays. Note: Cathode rays and canal rays are both shown by the vacuum tube devised by Professor C. T. Knipp, of the University of Illinois. See *Science*, Vol. XLII, page 942, December 31, 1915.

(*e*) Show luminescence produced by cathode rays. Common forms of Crookes tubes for this purpose are (1) A Crookes tube in which several varieties of minerals are mounted and (2) A Crookes tube in which the back of a metal butterfly is painted with several varieties of finely crushed minerals.

(*f*) Give a demonstration of X-rays using a fluoroscope. It is advisable for the operator to shield himself from X-rays by using screens of sheet lead.

(*g*) Show the ionizing action of X-rays by holding a Crookes tube so that the X-rays may pass near to the plate P', Fig. 58.

(*h*) Show the ionizing action of the α-rays, β-rays and γ-rays from radium by holding near P', Fig. 58, a metal plate upon which a minute quantity of radium has been deposited. This plate is to be kept in a small box when it is not in use.

(*i*) Exhibit the spinthariscope.

(*j*) Exhibit the paths in a fog chamber of individual α-particles from radium. A fog chamber for this purpose is for sale by The Cambridge Scientific Instrument Company, Cambridge, England.

PART IV.

LIGHT.

A NEW ENGINE, OR A HELP TO THE MIND CORRESPONDING TO TOOLS FOR THE HAND.

Bacon listed long ago, in his quaint way, the things which seemed to him most needful for the advancement of learning, and among other things he mentioned A New Engine, or a Help for the Mind corresponding to tools for the Hand; and the most remarkable aspect of present-day physical science is the aspect in which it constitutes a realization of Bacon's New Engine. We now force upon extremely meager data (obtained directly through our senses) interpretations which would seem to be entirely incommensurate* with the data themselves, and we exercise over physical things a kind of rational control which greatly transcends the native cunning of the hand. The possibility of this forced interpretation and of this rational control depends upon the use of two complexes.

(*a*) A *logical structure*, that is to say, a body of mathematical and conceptual theory which is brought to bear upon the immediate materials of sense, and

(*b*) A *mechanical structure*, that is to say, either a carefully planned *arrangement of devices* or a carefully planned *order of operations* such as the successive operations of solution, reaction, filtration, and weighing in chemistry.

These two complexes do indeed constitute a New Engine for the Mind, and the study of elementary physical science is intended to lead to the realization of this Engine (1) By the building up of the logical structure in the mind of the student, (2) By training in the use of devices, as in the making of measurements, and in the performance of ordered operations, and (3) By exercises in the application of (1) and (2) to the actual phenomena of physics and chemistry at every step and all of the time with every possible variation.

That surely is an exacting program; but the only alternative is to place the student under the instruction of Jules Verne where he need not trouble himself about foundations, but may follow his teacher pleasantly on a care-free trip to the moon or with easy improvidence embark on a voyage of twenty thousand leagues under the sea.

From *The Study of Science*, an introduction to our *Mechanics and Heat*, The Macmillan Co., 1910; reprinted in *Bill's School and Mine* by W. S. Franklin, published by Franklin, MacNutt and Charles, South Bethlehem, Pa.

* An astronomer, for example, *looks at a spec of light* as it crosses the field of his telescope, and he *listens to the ticks of a clock* to note the time of day when the spec crosses the center line of the field. He then *examines a set of fine lines on a divided circle* to find the angular altitude of the spec above the horizon. This he does three times in succession. Then he proceeds to calculate when the spec (a comet) will be nearest the sun, how far it will then be from the sun, how fast it will be moving, and when it will return, maybe a hundred years later!

LIGHT.

118. Unit sensory areas on the skin.—A very interesting experiment is to blind-fold an assistant and apply the points of a compass to finger tips, to back of hand, to arm and to the back of the neck, and bring the points of the compass closer and closer together until in each case the two points can no longer be distinguished as two.

The approximate diameter of the unit areas in the central part of the retina can be determined by placing two minute globules of mercury very close together (distance apart being measured) in bright sunlight, and finding greatest distance from the eye at which the two points of light are distinguishable as two points.

119. The astigmatic pencil of rays.—A clear understanding of the difference between the homocentric pencil of rays and the astigmatic pencil of rays is necessary if one wishes to understand the important subject of lens imperfections.

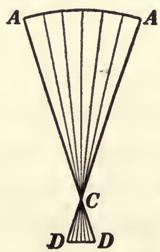

Fig. 61.
Heavy lines are stiff wires, light lines are fine strings.

A large wooden torus or ring is very helpful in the discussion of Fig. 276, page 357, *General Physics*. Also the wire model which is shown in Fig. 61 is very helpful. The wire AA forms an arc of a circle of which the center is at C. By rotating the model through a small angle about DD as an axis, the arc AA describes a portion of a ring surface; thinking of this surface as a wave front, the successive positions of the strings represent rays, and these rays intersect along the line C (perpendicular to the plane of the paper) and along the line DD.

Note.—A narrow astigmatic pencil of rays may be obtained by passing a beam of light from an electric arc obliquely through the central portions of a long-focus converging lens, as indicated in Fig. 277, page 358, *General Physics*, and the linear foci at O and at DD may be thrown upon a small piece of cardboard or on a translucent screen. The central portion, only, of the lens must be used, that is, the lens must be covered with an opaque disk with a hole at its center, otherwise the astigmatic effect is hidden by the complicated effects of the coma. See page 400, *General Physics*. A much broader astigmatic pencil, not perceptibly confused by coma, can be obtained by reflecting the light from an electric arc (or the light from the sun) obliquely from a concave mirror.

120. Total reflection.—The best demonstration of total reflection is to pass around the class several pieces of plate glass asking the students to note the silvery appearance of the squarely cut edge as seen by looking obliquely through the glass (showing how to look while making the statement); also several small 45°–45°–90° glass prisms may be used.

A brilliant beam of light may be passed through a tank of clear water and focused in a small aperture through which the water issues as a jet. The beam of light is kept in the smooth jet by total reflection until the surface of the jet becomes deeply rippled or breaks into drops. This experiment, which, it must be admitted, is of little value, may be modified by using a bent glass rod instead of the water jet.

121. Refraction of a spherical wave at a plane surface.—A very interesting and instructive experiment, which must be carried out by each student for himself, is as follows: The point O, Fig. 62, is a small bright object (a bit of chalk or a globule of mercury) on the blackened bottom of a basin of clear still water. Looking vertically downwards at O it appears to be at F (the center of curvature of the portion of the refracted wave $W'W'$ at or near V). Looking along the line aa DD,

the point O appears to be at DD if the line joining the two eyes is horizontal, but the point O appears to be at C if the line joining the two eyes is vertical. The pencil of rays at aa is astigmatic.

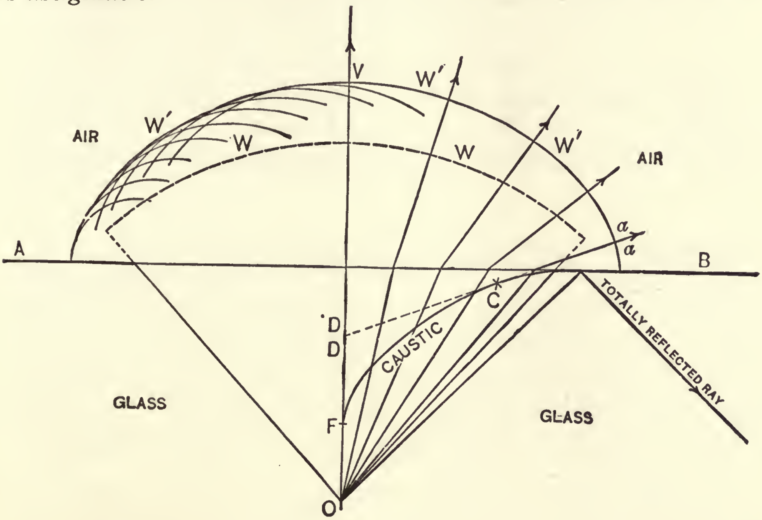

Fig. 62.

122. The actual lens and the ideal lens.—Pass a beam of light from an electric arc through a large, short-focus lens, such a lens, for example, as is used in the condenser of a projection lantern, and render the convergent beam of light beyond the lens visible by a cloud of chalk dust (using black board erasers) or by a cloud of ammonium chloride smoke. The appearance of the beam is shown in Fig. 342, page 397, *General Physics*, and described in Art. 287, page 397. This beam becomes very much more complicated if it passes obliquely through the lens.

Point out the fact that the whole theory of the simple lens as developed in Chapter XX, *General Physics*, applies to the ideal simple lens as explained in Art. 267, pages 373–374, *General Physics*.

123. Model of a compound microscope.—Mount as an object a small flat piece of fine wire gauze A, as object glass a one-inch

focal length lens O, and as an eye piece a longer focal-length lens E on a supporting bar as indicated in Fig. 63. Have this model in hand while explaining Art. 277, page 386, *General Physics*, and then pass it around the class.

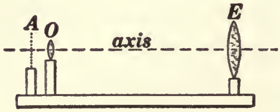

Fig. 63.

Fig. 64.

124. **Model of a simple telescope.**—Mount as object glass a long focal length lens O and as eye piece a shorter focal length lens E on a supporting bar as indicated in Fig. 64. Have this model in hand and use it to look at a distant object through an open window while explaining Art. 278, pages 387–388, *General Physics;* and give to each student an opportunity to use it.

125. **Experiment on vision.**—We base our sense of distance of an object (*a*) On a knowledge of the size of the object as compared with its apparent size, thus the finger at arm's length looks larger than, and can be made to cover, a man at a hundred yards. (*b*) By the muscular sense associated with the turning of both eyes directly towards the object, and (*c*) By the muscular sense associated with accommodation (see page 382, *General Physics*). If we look at an unfamiliar object with one eye through a pin hole held near the eye, bases (*a*) and (*b*) evidently fail, and base (*c*) also fails because accommodation is almost unnecessary when we look through a pin hole.

Stretch a cord across the room, using a kind of cord which comes in a wide range of sizes with same kind of twist. Looking at the cord with one eye through a pin hole walk towards it, then stop, and bring the fingers straight up from below as if to pinch the cord! The result is very amusing. The experiment works quite satisfactorily even when one has handled the cord and is familiar with its size. Sidewise or up-and-down move-

ments of the head tend to supply the missing sense of distance which is dependent on two-eye vision, and therefore the cord should be on a level with the experimenter's eyes and as the experimenter moves towards the cord he should not raise or lower his head nor move it sidewise.

The absence of any need for accommodation when one looks through a fine pin hole can be shown by looking through such a hole (held very close to the eye) at a printed page or preferably at a piece of wire gauze and bringing the gauze closer and closer to the eye.

126. Experiment on vision. The coin box.—Nothing, perhaps, is more familiar than a silver coin, and in a certain United States coinage one face is exactly similar in the dollar, the half-dollar, the quarter-dollar and the dime, and a very amusing device is a box with a fine peep hole at one end with these similar coins arranged at distances from the peep hole proportional to their respective diameters. The coins should be supported on pedestals the widths of which are in proportion to the diameters of the coins, and the pedestals should be high enough (box deep enough) so that the bottom of the box can be made invisible by a small screen at a short distance from the peep hole.

We, the authors, find that the coin box is almost sure to get out of order from one year to the next, because the coins get loose and are lost; but such things have never greatly disturbed us. We manage to give our best efforts to our students in spite of the ever insistent call of pessimism, and, as Professor Wm. Lyon Phelps says, teaching *is* great fun. But we know teachers, college teachers, who for such reasons, and often, alas, for reasons much less respectable and adequate, never take any pains at all! Professor Phelps has the advantage of us in that he teaches English. What unlimited opportunity for humor an English teacher has, and how shameful it is that English teaching should ever have the fatal quality of dullness! shameful even in the teacher of "crude Pennsylvania Dutch boys," who, according to one English teacher's comfortable belief, are unworthy of any

effort on his part!—We can say no more, lest "the gals hold their fans before their faces" as they did at Miss Meadow's party when Ole Brer Tarrypin led them to think that Brer Fox had cussed when he met him on the big road. Brer Fox hadn't cussed, he had merely said, Hello Stinkin Jim. Think of calling a nice man, like Brer Tarrypin, Stinkin Jim! His only fault is that he is constitutionally slow, but his patience is illustrated by what he tells Brer Fox about his being caught in a fire-swept field. Well, but what did you do Brer Tarrypin? Sot and tuck 'er, Brer Fox, sot and tuck 'er.

One of the most remarkable things in this world is the settled character of a teacher's position. Where a teacher begins he usually stays for life; and a teacher should make of his immobility an opportunity for virtue, like Ole Brer Tarrypin, even to the extent of teaching English to "crude Pennsylvania Dutch boys"; for all boys are crude; yes, even the far away groups that —that—never bother us, let us say.

127. Experiment on vision. Why is an object seen erect when its image on the retina is inverted?—In answer to this question the equally sensible question is sometimes asked: When one hears a baby cry with two ears, why does one not take it for twins? Our sensations are symbols which we interpret in accordance with experience.

Make a very small kink or curl at the end of a bit of very fine wire, hold the curl near the eye, beyond the curl hold a fine pin hole and look towards a broad bright surface. An erect shadow (an erect image) of the curl is thrown on the retina, and the curl is seen inverted. If we always looked at curls in this way we would develop the habit of seeing them right side up.

The compound microscope and the astronomical telescope always invert an object which is being examined, and this inversion is confusing to a novice, but to the experienced astronomer or microscopist it is not in the least confusing or even troublesome.

128. Experiment on vision. Curious effect of two-eye vision. —Stretch a thread or string on a frame as indicated in Fig. 65, and look at the string with the two eyes as indicated by the arrow. Bring the eyes to focus on any point p of the string and one will see two strings intersecting at the point p.

Fig. 65.

129. Shadows of blood veins on the retina.—Any persistent phase of retinal excitation which is independent of external objects and conditions is by long habit ignored and therefore imperceptible. For example, a complex system of arteries and veins lies in front of the retina and casts shadows on the retina, but ordinarily these shadows are not perceptible. However, if the image of a kerosene lamp flame is thrown by a moderately short focus lens on the eye ball (eye being turned away from direction of lamp so that image may fall on the eye ball at some distance to one side of the cornea) in a darkened room, the small quantity of light which penetrates the thick walls of the eye casts shadows of arteries and veins in an unusual manner and the entire net-work of veins and arteries becomes visible.

130. Persistence of vision and the stroboscope.—The light sensation produced by a flash of light lasts from one half of a second to one second or more, according to the intensity of the light, and an intermittent light is sensibly continuous when the flashes recur at a frequency of 15 or 20 flashes per second or more.

Move the hand rapidly to and fro, with outspread fingers, in the light of an alternating-current electric arc, and many duplicate images of the hand are seen because of the intermittent character of the light from such an arc.

Mount a slotted metal disk on the shaft of a small electric motor and arrange an electric arc (direct-current arc) near the edge of the disk so that flashes of light may pass through the slot or slots as the disk rotates. A cardboard disk with figures

on it marked in broad black lines like Fig. 66 is mounted on a second motor, the cardboard disk is illuminated by the flashes of light, and the speed of one or both motors is adjusted until the ratio of the motor speeds is an integer. With the particular marking shown in Fig. 66 the widest range of effects can be obtained if the slotted metal disk has four equidistant slots, and with a 10 or 12-inch disk the slots should be about $\frac{1}{2}$ an inch wide.

Fig. 66.

A most amusing experiment is to use a cardboard disk marked as indicated in Fig. 67.

Fig. 67.

131. Reversal of a sense of fatigue.—In many cases it seems that an even blend of excitation of many nerve elements gives, let us say, a neutral or colorless sensation, whereas the predominance of excitation of certain elements of the blend gives a sharp and distinctive character to the corresponding sensation. This is exemplified by the sensations of color according to the Young-Helmholtz theory (see *General Physics*, pages 479–481), and it

is also shown by the reversal by fatigue of the sense of motion. When one looks at the floor after looking for a long time out of the window of a moving railway car, one sees the floor in motion in a reversed direction. Mark a broad black spiral on a cardboard disk, rotate the disk steadily at a speed of three or four revolutions per second, look intently at the rotating disk for half-a-minute or more, and then turn and look at the face of a nearby companion. Your companion's head will be seen to swell or shrink according to the direction of motion of the disk and spiral.

This reversal of the sense of direction by fatigue seems to show that all the nerve excitation which is necessary for a sense of motion in any direction is present in a complex blend when we look at a stationary field; but that steady gazing at a field moving in one direction produces fatigue which results in the weakening of a certain element of excitation in what would ordinarily be a neutral or colorless blend when we look again at a stationary field.*

132. Imperfection of focal spot of a perfect lens.—By passing the light from a distant electric arc through a very fine pinhole *p*, Fig. 68, and looking through the magnifying glass *M*, the distribution of the light on the plane *FF* can be seen. If the pin hole is very fine, the distribution of light gives an exaggerated example of the imperfections of the focal spot of a perfect lens as explained in Art. 283, pages 392–394, *General Physics*.

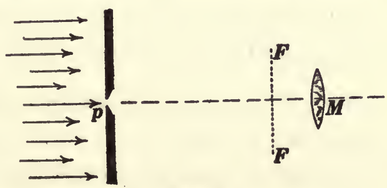

Fig. 68.

133. Axial spherical aberration.—See experiment 122.

134. Astigmatism of a lens of narrow aperture.—(*a*) See note under experiment 119.

* This matter is discussed in a brief note in *Science*, Vol. IX, page 70, Jan. 13 1899.

(*b*) The astigmatism of a lens may be shown by looking obliquely through a low power magnifying glass at the cross-rulings on cross-section paper, the lens being held so that its central plane intersects the plane of the paper in a line parallel to one set of rulings. In this case the pupil of the eye limits the effective beam of light to a very narrow pencil, and one or the other sets of cross rulings may be sharply focused by adjusting the distance of the lens from the paper.

(*c*) Substitute a simple lens for the object lens of a projection lantern, cover the lens with an opaque disk with a hole at its center, turn the lens so that the light passes obliquely through it, and use a piece of wire gauze as a lantern slide. By adjusting the distance of the oblique simple lens from the gauze the vertical or horizontal wires may be focused on the lantern screen. Of course the central plane of the oblique lens must intersect the plane of the gauze along a line parallel to one set of wires.

135. Image distortion.—Using a piece of wire gauze as a lantern slide, and a simple lens as the object lens of the lantern, the two kinds of image distortion can be shown as explained in Art. 291, pages 402–403, *General Physics*. It is worth while also to use the regular orthoscopic lantern objective and to project an undistorted image of the gauze on the lantern screen.

136. Chromatic aberration.—The image of the wire gauze as projected by the simple lens in experiment 135 shows traces of color due to the chromatic aberration of the lens, but the beam of light passes chiefly through the central part of the lens as indicated in Figs. 352 and 353, page 403, *General Physics*, so that the chromatic aberration is small as explained in connection with Fig. 362, page 411, *General Physics*.

The regular orthoscopic lantern objective is corrected for achromatic aberration and it projects a nearly color-free image of the wire gauze on the screen.

The lenses of the eye are by no means achromatic, but we have come by habit to neglect, and therefore not to see, the color

fringes even of brilliant objects. Any unusual condition, however, makes this habit ineffective. Thus by looking at a distant brilliant lamp a very decided show of color is seen as the finger is held near the eye and moved slowly sidewise until it nearly covers the pupil.

An experimental study of chromatic aberration suitable for the laboratory is described in Franklin, Crawford and MacNutt's *Practical Physics*, Vol. III, The Macmillan Co.

137. The telephoto lens.—Figure 69 represents the object lens O and the eye piece E of a simple telescope, the eyepiece being drawn out so that its distance from the image i is greater than

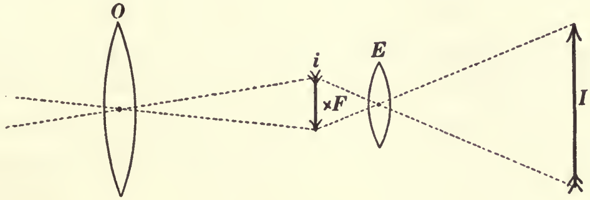

Fig. 69.

its focal length EF. Under these conditions an enlarged real image of i is formed at I. Figure 70 represents an opera-glass type of telescope with the eyepiece E pushed in so that its distance from the image i is greater than its focal length EF. Under these conditions the eyepiece forms an enlarged real image of i at I. Either arrangement, Fig. 69 or Fig. 70, can be used as a telephoto lens but the arrangement in Fig. 70, being shorter, is more convenient. The individual lenses O and E are always compound lenses. Thus the left-hand group of lenses in Fig. 373, page 417, *General Physics*, takes the place of O in Fig. 70, and the right-hand group of lenses in Fig. 373 takes the place of E in Fig. 70.

An experiment of great interest is to use a simple telescope as indicated in Fig. 69 for projecting an image of the sun on a sheet of cardboard. If a large astronomical telescope is not

available a small reading telescope from the laboratory will answer. The telescope should be fixed in a hole through a

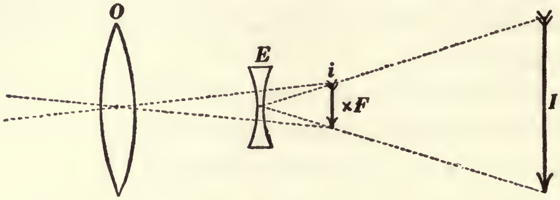

Fig. 70.

large board (to darken the region back of the telescope), and the image should be thrown on a translucent screen of ground glass so as to be easily seen by a group of persons.

138. **Spectroscope demonstrations.**—To project a large and sharply defined spectrum upon a screen requires very expensive apparatus and therefore the authors always arrange several laboratory spectroscopes so that the members of a class may one by one see the following:

(*a*) The continuous spectrum of a gas flame or glow lamp.

(*b*) The bright-line spectrum of an alkali metal in a Bunsen flame.

(*c*) The reversal of the sodium lines, and

(*d*) The dark-line spectrum of the sun.

The reversal of the sodium lines may be seen by looking through a Bunsen flame at the crater of a direct-current carbon arc and placing in the Bunsen flame a small roll of asbestos cloth which has been previously soaked in strong brine and dried.

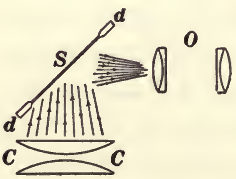

Fig. 71.

139. **Projection of a soap film.**—The interference colors of a thin film may be beautifully shown by projection. A thin metal disk *dd*, Fig. 71, with a circular hole two or three inches in diameter is dipped into a soap solution and placed as shown in front of the lantern condenser *CC*

and an image of the soap film S is projected on the screen by the lantern objective O.

140. Spectroscopic analysis of the light reflected from a thin film.—Arrange a thin film of mica so as to reflect the light from a gas flame or glow lamp into the slit of a spectroscope. It is desirable in this experiment to use a film too thick to show interference color to the unaided eye. The spectrum will show a large number of bright and dark bands, and no color is seen directly by the unaided eye because the intensified wave lengths (and the weakened wave lengths) are distributed throughout the spectrum. When light from a soap film is reflected into the slit of a spectroscope one or two parts of the spectrum are intensified (and one or two parts weakened) at a time.

141. The diffraction grating.—A most striking demonstration is to look through a glass grating (200 or 300 lines to the inch) at an electric arc between impregnated carbons. This gives the arrangement shown in Fig. 400, page 440, *General Physics*, LL being the lens of the eye and PP being the retina.

142. Polarized waves on a rubber tube.—Tie one end of a rubber tube fast and hold the other end in the hand with the tube stretched, and give the discussion of Art. 311, page 442, *General Physics*. Pass the rubber tube through two narrow slits, produce irregular waves on the tube and point out the polarizing effect of the slit near the hand. Turn the second slit so as to be parallel to the first and then turn it so as to be at right angles to the first and show the effects as described.

143. Group of experiments on polarized light.—Polarizing apparatus for projection is expensive and the authors have therefore always arranged what our students have called a "seven-ring circus" for demonstrations in polarized light. The demonstrations we use in spectroscopy also constitute a "seven-ring circus." Each set of apparatus is usually duplicated, and the entire teaching staff of the department usually serve as "ring-masters."

(*a*) Show tourmaline tongs and direct the observer to look through the tongs and turn one of the crystals slowly.

(*b*) Look through a Nicol prism at a bright varnished surface and turn the prism slowly.

(*c*) Place a small rhomb of Iceland spar on a sheet of paper on which is a small black dot and turn the rhomb slowly. Look at the rhomb and dot through a Nicol prism and turn the prism slowly.

(*d*) Look through two Nicol prisms and turn one of the prisms slowly.

(*e*) Using a simple polariscope (parallel beam) examine a plate of mica or other doubly refracting crystal, turning polarizer and crystal plate slowly, one and then the other.

(*f*) Using a polariscope (arranged for convergent beam) examine a plate of Iceland spar cut perpendicularly to optic axis; also examine a plate of a biaxial crystal, aragonite for example.

(*g*) Using a simple polariscope (parallel beam) examine a cube of glass under stress.

Note.—The inexpensive Norrenberg polariscope can be used for *e* and *g*; and for *f* a pair of tourmaline tongs held close to the eye is perhaps most satisfactory.

Note.—Experiment *c* can be easily projected. Use a metal lantern slide with a pin hole in it, and place a rhomb of Iceland spar against the slide covering the pin hole, and turn the rhomb slowly.

Note.—A large cardboard model of a rhomb of Iceland spar with equal length edges and with a rod set in the position of the optic axis (the axis of symmetry)is useful.

Note.—A large block of wood cut to the form of a Nicol prism and sawed in two like a Nicol prism is useful.

Note.—The following experiments are very interesting but inconvenient and unsatisfactory for general purposes.

(*h*) Fill a tall glass jar with a milky solution made by dropping

a few drops of extremely dilute rosin-alcohol into water and stirring vigorously. Throw a beam of light downwards into the jar in a dark room and look sidewise at the jar through a Nicol prism, turning the prism slowly. Throw a beam of plane polarized light downwards into the jar, rotate the plane of polarization, and look at the jar from one side. In this second experiment a test tube may be used instead of a jar so that a small sized Nicol prism can be used as a polarizer.

(*i*) Fill a test tube with a strong solution of syrup made milky by the addition of a small quantity of extremely dilute rosin-alcohol. Throw a beam of plane polarized light downwards into the tube in a dark room and look at the tube from one side. The tube will show faint helices of colored bands.

Note.—The authors have always arranged for students to use the saccharimeter in the laboratory. See Franklin, Crawford and MacNutt's *Practical Physics*, Vol. III, pages 68–73.

144. Group of experiments on color.—

(*a*) Light from a gas flame or glow lamp passes into the slit of a spectroscope and the observer places before the slit in succession a series of colored glasses or a series of colored solutions in narrow glass cells.

(*b*) Light from a gas flame or glow lamp is reflected into the slit of a spectroscope from a series of colored pieces of paper or cloth. It is advisable to have a piece of white paper placed permanently in position to reflect light into the spectroscope when the colored paper or cloth is removed.

(*c*) Illuminate a batch of brightly colored objects (a selection of small skeins of brilliantly colored worsted such as is used for color-blind tests) by the light from a sodium flame in a dark room and arrange an electric glow lamp so that it can be easily turned off and on so as to give white light illumination.

(*d*) Demonstrate the familiar rotating sector-colored disk, and adjust the sectors so as to give a neutral gray resultant or blend.

(*e*) Contrast effect. Place side by side on an inclined board a large sheet of green tinted paper and a large sheet of red tinted paper with good illumination. Tear off two bits of grayish paper (from the same piece) placing one of the bits on each sheet of tinted paper, and tear off a bit of each sheet of tinted paper and place the bit upon the other sheet. The colors of the small bits of paper are so modified by the effects of contrast that it is almost impossible to believe that the two bits of grayish paper are the same, and to believe that each bit of colored paper is the same as the nearby large sheet. This experiment is most effective if the bits of paper are torn off and placed after calling the students' attention to the experiment and while they are looking attentively.

(*f*) The authors usually arrange for color-blind tests of the entire class. We ordinarily use the Holmgren test,* and usually we can persuade a color-blind person to undergo the test before the entire class.

* Colored worsteds for this test may be obtained from E. B. Meyrowitz, N. Y. City.

PART V.

SOUND.

THE PHILOSOPHY OF STEAM SHOVELS AND THE PHILOSOPHY OF LIVING.

Imagine a never-to-be-escaped human need of a twenty-foot arm! What age-long development and what unthinkable pains! It is easier to build a steam shovel. All of which means that *homo sapiens* is now bent towards Social Inheritance; but social inheritance has own pains as many know who burn the midnight oil.

How shocking to reduce the tender-minded philosopher's love of perfect precision* to a materialistic preference for steam shovels as opposed to immeasurable pains of birth; and to make mathematical philosophy appear as a dire necessity† rather than as a thing to be chosen for its own sake. And then to urge‡ with that lover of paradox, Gilbert Chesterton, that the serious spiritual and philosophic objection to steam shovels is not that men work at them and pay for them and make them very ugly, nor even that men are killed by them, but merely that men do not play at them! Think of a group of sportsmen cavorting over a ten-thousand acre field tossing and catching a Brobdignagian ball in steam shovels! It *is* conceivable that the one objection to the steam shovel might have been overcome if the Great War had not come upon us.

The great danger of our time has been the confusion of boundaries between thing-philosophy and human philosophy, between the philosophy of material conquest and power and that intimate philosophy of comfort which makes life not easy but worth while. When these boundaries are rectified there will be one philosophy of steam shovels, recognized and used as such, and another philosophy of living. Science will then stand as the essence of man's inescapable responsibilities in practical affairs, and we shall seek God, a finite God, in that which is intimately and even narrowly human, if narrowness there be in that supreme and illimitable mystery.

From Education After the War by Franklin and MacNutt; reprinted in *Bill's School and Mine.*

* This is a reference to the point of view of Professor Keyser of Columbia University as hazily set forth in his recent articles On the Human Worth of Precise Thinking. See page 34 of this volume for further comment.

† See our Introduction to Mechanics (The Study of Science), as referred to on page 140 of this volume.

‡ See Preface to our *Elements of Electricity and Magnetism*, The Macmillan Co., 1908.

SOUND.

The senses of touch and sight are preëminently space senses, as everyone knows, whereas the sense of sound might almost be called the time sense, because the order in sequence of the elements of a given sound sensation is all-important. This is true of fleeting, irregular sounds as in speech, and it is equally true of musical sounds as is shown by the use of rythm and sequences of tones in music.

145. **An extremely amusing experiment,** and one which illustrates the importance of order or sequence in the elements of a complex sound sensation, is to arrange a phonograph so that it can be driven forwards or backwards at will and produce, forwards and backwards, a familiar melody like "Yankee Doodle" or a familiar speech like "Mary Had a Little Lamb."

A melody or a speech when reversed produces, of course, precisely the same sense elements but nothing is more utterly unlike than a melody and its reverse or a speech and its reverse.

146. **Musical sticks.**—To show that musical tones commonly are component parts of many noises thump at different points on a table or chair and call attention to the fact that the sound differences are mostly differences of pitch in the very brief tones that are produced.

Drop a stick on the floor and the sound is mostly noise, but drop a carefully adjusted series of sticks and the tone differences stand out by contrast as a clearly recognized melody or musical scale.

147. **The Galton whistle.**—An interesting experiment for a very small group of listeners is to determine the pitch limit of audibility of musical tones by means of the Galton whistle.

A series of steel bars giving tones in an ascending scale up to

and beyond the limit of audibility can be used. These bars can be cut from cold rolled steel shafting and if d is the diameter and l the length of a bar in inches the number of complete vibrations per second (transverse vibrations) corresponding to its fundamental mode is

$$n = 15010 \frac{d}{l^2}$$

148. Simple modes of vibration of a string.—Produce the successive harmonics on the string of a violin or guitar. Using a sonometer place paper riders on the string and show the experiment which is described at the top of page 509, *General Physics.*

149. Simple modes of vibration of an air column.—Produce the successive notes on a bugle. Using a long narrow organ pipe blow it with increasing air pressure, and a series of tones corresponding to successive simple modes will be clearly heard.

A long glass tube an inch or an inch and a half in diameter is arranged as a whistle or organ pipe, closed at one end and blown steadily by bellows (the moist breath will not do), and the vibrating segments are made visible by means of lycopodium powder. Increasing the air pressure will cause the air column to break up into shorter and shorter vibrating segments.

150. Simple modes of vibration of a plate. Chladni's figures.—The simple modes of vibration of a plate may be shown as explained on pages 517–518, *General Physics.*

151. Resonance.—(*a*) Free the strings of a piano by pressing the damper pedal and sing a series of clear notes loudly with the mouth held near to the sounding board. Each note will set in vibration that particular piano string which vibrates in unison with it, and each note will be heard (as produced by the vibrating string) when the sung note ceases. A piano is an important part of the equipment of a physics lecture room.

(*b*) Hold a vibrating tuning fork over the mouth of a tall

slender glass jar, and pour water slowly into the jar. The air in the jar will be set vibrating by the fork when the frequency of vibration of the air column coincides or nearly coincides with the frequency of the vibrating fork and the sound produced will be greatly intensified. This experiment is most satisfactory when the water flows into (or out of) the tall jar quietly through a rubber tube connection at the bottom of the jar.

(*c*) An ordinary telephone connected to alternating-current supply mains (through a high resistance, of course) produces a musical tone which is very rich in over-tones, and if such a singing telephone is held near the mouth of the tall jar above described the successive over-tones come out with wonderful distinctness as the jar is slowly filled with water. The over-tones may be brought out also by holding the singing telephone near one's open mouth and changing the mouth cavity as if to speak the following vowels in succession ṳ, ō, a̤, ā, ĕ and ē (see General Physics, page 522). Ordinary 60-cycle alternating current is not very satisfactory for use in this experiment, but the experiment is wonderful if one has 133-cycle or 150-cycle current. One can produce sufficient alternating current by an electrically driven tuning fork, the telephone (very low resistance telephone) being connected between the terminals of the windings of the driven magnet, and, if necessary, a large capacity condenser being connected across the break. A tuning fork of 250 or 300 complete vibrations per second should be used. An ordinary pitch pipe can be used instead of a singing telephone but the overtones do not come out very sharply.

152. Experiments on vowel sounds.—Shape the mouth as if to produce the vowels u, ō and a̤ in succession and thump the cheek, and the characterizing tones (173, 517 and 775 vibrations per second, respectively, see page 522, *General Physics*) will be distinctly heard.

An extremely amusing experiment is to fill the lungs with carefully purified hydrogen and repeat deliberately a familiar speech like "Mary Had a Little Lamb," or speak deliberately

the following words in order: *rude*, *no*, *paw*, *part*, *pay*, *pet* and *see*, as explained on page 523, *General Physics*. The use of familiar words which are recognizable by the accompanying consonants tends to correct one's perception of the false vowel sounds in spite of any effort of the will on the listener's part. Therefore it is better to pronounce the successive vowels ṳ, ō, a̤, ä, ā, ĕ and ē, and point each time to a word (written on the blackboard) which contains the particular vowel.

Note.—Hydrogen made by dissolving zinc in sulphuric acid is apt to contain arsenic hydride. Purify by passing the hydrogen through a strong solution of potassium permanganate.

153. Illustration of the action of the ear in the perception of tone quality or timbre.—Free the strings of a piano and sing a vowel sound loudly against the sounding board as described and explained in Art. 373, page 524, *General Physics*.

154. Beats.—Blow two similar organ pipes together and alter the pitch of one of the pipes slightly by means of a paper or cardboard extension of the open end of one of the pipes.

155. Combination tones.—A combination tone (difference tone) which can be heard throughout a large room can be produced as explained on page 528, *General Physics*.

PART VI.

THE DYNAMICS OF WAVE MOTION.

The substance of this entire part VI was given in a lecture by Wm. S. Franklin before a joint meeting of the Western Society of Engineers and the Chicago Section of the American Institute of Electrical Engineers on May 28, 1917. Incredible as it may seem, nearly all of the mathematics here given and all of the experiments here described were included in the lecture.

SCIENCE AND TECHNOLOGY VERSUS THE HUMANITIES IN EDUCATION.

The worst cant of our time, touching the Idolatry of Science*, which is our sincerest religion, and handled to perfection by our easier college product, is the semi-serious wail of regret that a silver-spoon smartness was not transmuted by a pleasant college course into Knowledge and Appreciation of Science. Knowledge and Appreciation—alas! It were better to say *easy acquisition* instead of appreciation, and *fabulous riches* instead of knowledge. Riches in which nearly every man has had a share, unearned.

Unearned, indeed, and yet much more than earned! Every loafer knows something of the unpleasant exactions of effective labor, but there never yet was a dilettante who could even dream of the pains of those who really learn nor any self-satisfied Philistine who could sense the grief of those who are wise!

What do you think we are driving at with all this apparent rhetoric? Merely to express the mood we fall into when we read such things as Paul Shorey's "The Assault on Humanism" (see page 34 of this volume for further comment) and think of the cockey attitude of many of our friends who have to do with technical education. What our friends need is to be obliged to defend their ground before a jury of Paul Shoreys, and what the Paul Shoreys need is to hear us confess how little we are able to accomplish in our teaching of the mathematical sciences. But much as we approve of Paul Shorey's point of view we are very far from capitulating to the easy talk of educationalists concerning the real aim of education as the development of personality and character.

Our greatest comfort as teachers comes from the exaggerated idea among those who do not teach (and among others who only pretend to teach) as to what can be accomplished by teaching. That teacher of philosophy, we know that he accomplishes but little however bigity his talk. The manufacturer who prates about character, he knows what he is talking about all right, from his point of view, but his talk gives us no distress—at least no distress concerning our work. And we know that personality and character cannot be developed by sentimentality, however articulate. Science and technology versus the humanities; the antithesis does not disturb us, we are at war with the Philistines!

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* Contempt for science has been much in evidence in England and America in the past, and of recent years science has been more and more idolized. With no abatement of Contempt, an Idolatry of science has developed, and these two extremes are blended together in the same men!

Science is *Finding Out* and *Learning How*, but most men think of science in terms of results. These results have fascinated the crowd, and the great majority of men "have adopted a scale of physical values for everything in life with a consequent neglect of quality and a denial of human value in everything. We have a philosophy of rectangular beatitudes and spherical benevolences, a theology of universal indulgence, a jurisprudence which will hang no rogues; all of which means, in the root, incapacity of discerning worth and unworth in anything and least of all in man. Whereas, Nature and Heaven command us at our peril, to discern worth from unworth in everything and most of all in man.

THE DYNAMICS OF WAVE MOTION.

One of the most important items of needed improvement in the curriculum of the engineering school is the rejuvenation of the usual, dried-up and unfruitful course in theoretical mechanics, and this discussion of the simplest aspects of the dynamics of wave motion is intended to show what can be done to make this important branch of mechanics intelligible to the engineering student. As Heaviside says, the physics of wave motion is extremely simple and the mathematics extremely difficult for the wave pulse, whereas the physics is extremely complicated and the mathematics extremely simple for the sine-wave train. Therefore the consideration of periodic waves* is almost entirely excluded from this discussion.

The important subject of wave-diffusion or wave-distortion is discussed in an extremely simple manner in Arts. 121–123 (pages 222–228) of Franklin and MacNutt's *Advanced Electricity and Magnetism*, The Macmillan Co., 1915.

The exhaustive mathematical treatment of wave-distortion (due to resistance and leakage of a telephone line, for example) is based almost entirely on considerations such as are involved

* A well-known elementary treatise on physics, before making use of the wave theory of light "proves" that light is wave motion, as follows: Any effect which is periodic and which travels at finite velocity is wave motion; light is periodic and it travels at finite velocity; therefore light is wave motion! The idea of the Gatling gun popped into our minds when we first came across this argument and the effect of the Gatling gun is periodic and is propagated at finite velocity! In fact periodicity of waves in an extended medium always depends upon and is determined by actions which take place in the system which produces or absorbs the waves (water waves produced by wind of a given velocity constitute an exception), and waves have, in general, no definite velocity, as everyone knows who understands what Heaviside calls wave-diffusion or who understands the difference between *group velocity* and *wave velocity* in Helmholtz's theory of dispersion. The fact is that the theory of periodic waves lends itself easily to unlimited algebraic formulation and few men know anything beyond these formulas and the pictures of sine curves which underlie them!

in the discussion of the alternating-current transmission line on pages 141–153 of W. S. Franklin's *Electric Waves*, The Macmillan Co., 1909. This discussion is unique in the simplicity with which the troublesome boundary conditions are set forth in Figs. 132, 140–147.

The equation of a traveling curve.—The curve cc, Fig. 72, is stationary with respect to the origin of coördinates O', and the curve and the origin O' are assumed to be traveling together towards the right at velocity v so that the abscissa of the

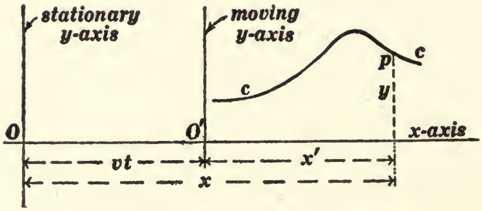

Fig. 72.

moving origin O' as referred to the fixed origin O is at each instant equal to vt as indicated in the figure. Therefore the abscissa x of the point p on the moving curve is $x = x' + vt$, so that $x' = x - vt$. Let the equation to the moving curve cc as referred to the moving origin O' be

$$y = F(x')$$

Then, substituting $x - vt$ for x', we have

$$y = F(x - vt) \tag{1}$$

as the equation of a curve traveling to the right at velocity v; and in a similar manner it may be shown that

$$y = f(x + vt) \tag{2}$$

is the equation of a curve traveling to the left at velocity v.

It is highly instructive to derive a differential equation which

is satisfied by both equations (1) and (2); and, inasmuch as the rules for differentiating are not usually understood by the student, especially when applied to such general equations as (1) and (2), the following discussion is arranged to appeal to one's fundamental arithmetical sense. Awkwardness of notation is the chief difficulty, as usual, and one must remember the following points:

If y increases always u times as fast as x, then u is called the *derivative of* y *with respect to* x. If y depends on x as the only variable, the derivative is represented by the symbol dy/dx. If y depends on more than one variable, the derivative with respect to x is called a *partial derivative* and it is usually represented by the symbol $\partial y/\partial x$. The two symbols dy/dx and $\partial y/\partial x$ have, however, precisely the same arithmetical meaning; thus in the first case y actually increases dy/dx times as fast as x, and in the second case y *would* increase $\partial y/\partial x$ times as fast as x if x only, among all the variables upon which y depends, were allowed to change.

If u $(= dy/dx$ or $\partial y/\partial x)$ increases w times as fast as x, then, of course, w is the derivative of u with respect to x and of course it is represented by du/dx or $\partial u/\partial x$. In thinking of the derivative of u, however, it is often desirable to keep clearly in mind the fact that u itself is the derivative of y with respect to x. This is shown by the following notation:

$$\frac{du}{dx} = \frac{d^2y}{dx^2} \quad \text{when} \quad u = \frac{dy}{dx}$$

$$\frac{\partial u}{\partial x} = \frac{\partial^2 y}{\partial x^2} \quad \text{when} \quad u = \frac{\partial y}{\partial x}$$

Partial differential equation of travel.—The two equations (1) and (2), above, are particular solutions of a partial differential equation which may be called the *partial differential equation of travel*, namely

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2} \tag{3}$$

and this differential equation is of fundamental importance in the mathematical theory of wave motion.

For the sake of simplicity let the quantity $(x - vt)$ of equation (1) be represented by z so that

$$z = x - vt \tag{i}$$

from which we have $\partial z/\partial x = 1$ (which means that if x, only, increases z must increase at the same rate), and $\partial z/\partial t = -v$ (which means that if t, only, increases z must increase $-v$ times as fast or decrease v times as fast).

Equation (1) is to be thought of for the moment as $y = F(z)$, and the derivative dy/dz may be represented by $F'(z)$, meaning that y increases $F'(z)$ times as fast as z.

Proposition: y increases $F'(z)$ times as fast as z, and when x, only, changes z increases $\partial z/\partial x$ times as fast as x. Therefore when x, only, changes y must increase $F'(z)$ *times* $\partial z/\partial x$ *times* as fast as x. Therefore $\partial y/\partial x = F'(z)\cdot(\partial z/\partial x)$. But $\partial z/\partial x = 1$ as stated above. Therefore*

$$\frac{\partial y}{\partial x} = F'(z) \tag{ii}$$

Proposition: y increases $F'(z)$ times as fast as z, and when t, only, changes z increases $\partial z/\partial t$ times as fast as t. Therefore when t, only, changes y must increase $F'(z)$ *times* $\partial z/\partial t$ *times* as fast as t. Therefore $\partial y/\partial t = F'(z)\cdot(\partial z/\partial t)$. But $\partial z/\partial t = -v$ as stated above. Therefore

$$\frac{\partial y}{\partial t} = -v\cdot F'(z) \tag{iii}$$

From equations (ii) and (iii) it is evident that $\partial y/\partial t = -v\cdot(\partial y/\partial x)$, and this may be called the differential equation of travel to the right. If we had started with $y = f(x + vt)$

* To the student who is familiar with the rules for partial differentiation this argument may seem ridiculous, but it is not ridiculous, by any means. No one can understand partial differenfiation who has not at some time followed this kind of an argument in pure arithmetic.

and $z = x + vt$, the above discussion would have led to $\partial y/\partial t = +v\cdot(\partial y/\partial x)$, which may be called the differential equation of travel to the left. Neither of these differential equations is of importance in the theory of wave motion.

Let us think of the function $F'(z)$ as changing $F''(z)$ times as fast as z.

When x alone, changes, z changes $\partial z/\partial x$ times as fast as x, and therefore $F'(z)$ changes $F''(z)$ *times* $\partial z/\partial x$ *times* as fast as x. But $F'(z)$ is equal to $\partial y/\partial x$ according to equation (ii). Therefore $\partial y/\partial x$ increases $F''(z)$ *times* $\partial z/\partial x$ *times* as fast as x. Therefore, since $\partial z/\partial x = 1$, we have

$$\frac{\partial^2 y}{\partial x^2} = F''(z) \tag{iv}$$

When t alone, changes, z changes $\partial z/\partial t$ times as fast as t; therefore $F'(z)$ increases $F''(z)$ *times* $\partial z/\partial t$ *times* as fast as t, and therefore $-v\cdot F'(z)$ increases $-v$ *times* $F''(z)$ *times* $\partial z/\partial t$ *times* as fast as t. But $-v\cdot F'(z)$ is equal to $\partial y/\partial t$ according to equation (iii). Therefore $\partial y/\partial t$ increases $-v \times F''(z) \times \partial z/\partial t$ times as fast as t, and therefore, since $\partial z/\partial t = -v$, we have

$$\frac{\partial^2 y}{\partial t^2} = v^2 F''(z) \tag{v}$$

From equations (iv) and (v) we have

$$\frac{\partial^2 y}{\partial t^2} = v^2\cdot\frac{\partial^2 y}{\partial x^2} \tag{3}$$

and this same result would be obtained from equation (2) by taking $z = x + vt$. That is to say, equation (3) is a differential equation which is satisfied by both of the equations (1) and (2), and it is of very great importance in the mathematical theory of wave motion.

The principle of superposition. A property of linear differential equations.—A principle of extremely wide application in physics is the so-called principle of superposition. From the

physical point of view a general statement of the principle is scarcely possible, and therefore the following examples must suffice: (1) A person at A in Fig. 73 can see window No. 1 and another person at B can see window No. 2 *at the same time*. This means that the two beams of light a and b can travel through the same region at the same time without getting tangled together, as it were; each beam behaves as if it were traveling through the region alone. (2) Two sounds can travel through the same body of air simultaneously, and each sound travels as though it occupied the space by itself. (3) Two systems of water waves can travel over the same part of a pond simultaneously, each system behaving as if the other were not present. (4) Two messages* can travel over a telegraph wire simultaneously and not get mixed up together. (5) Two forces F and G exerted simultaneously upon an elastic structure produce an effect which is the sum of the effects which would be produced by the forces separately, provided the sum of the forces does not exceed the elastic limit of the structure, therefore each force may be thought of as producing the same effect that it would produce if acting alone.

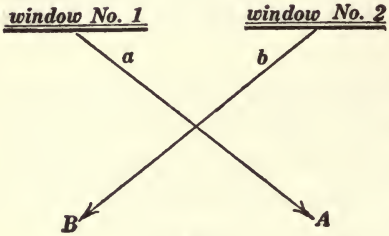

Fig. 73.

All of the effects in physics which are superposable—and this includes by far the greater portion of the effects in mechanics, heat, electricity and magnetism, light and sound, and chemistry —are expressible in terms of linear differential equations, and the principle of superposition is a clearly defined property of such equations as follows: *If* y *is a function of* x *which satisfies*

* Indeed any number of distinct messages can travel over a telegraph wire in either direction or in both directions simultaneously. The only limiting feature in multiplex telegraphy is the design of the sending and receiving apparatus; and the same is true in wireless telegraphy. In each of the above examples the word *two* means *two or more.*

a linear differential equation, and if z *is another function of* x *which satisfies the same differential equation, then* $(y+z)$ *is a function which satisfies the differential equation.*

This proposition is true for both ordinary and partial linear differential equations, and indeed nearly all of the superposable effects in physics are expressible in terms of partial linear differential equations. The proof of the proposition is, however, nearly the same for ordinary and for partial differential equations, and therefore it is sufficient to give the proof for ordinary differential equations only. Let the given linear differential equation be:

$$u + A\frac{du}{dx} + B\frac{d^2u}{dx^2} + \cdots = 0 \qquad \text{(i)}$$

Let y be a function of x which satisfies this differential equation, then:

$$y + A\frac{dy}{dx} + B\frac{d^2y}{dx^2} + \cdots = 0 \qquad \text{(ii)}$$

Let z be another function of x which satisfies (i), then:

$$z + A\frac{dz}{dx} + B\frac{d^2z}{dx^2} + \cdots = 0 \qquad \text{(iii)}$$

Now

$$\frac{d(y+z)}{dx} = \frac{dy}{dx} + \frac{dz}{dx} \quad \text{and} \quad \frac{d^2(y+z)}{dx^2} = \frac{d^2y}{dx^2} + \frac{d^2z}{dx^2}$$

Therefore, adding equations (ii) and (iii), we get:

$$(y+z) + A\frac{d(y+z)}{dx} + B\frac{d^2(y+z)}{dx^2} + \cdots = 0 \qquad \text{(iv)}$$

But equation (iv) is exactly the same form as equation (i), and therefore $(y+z)$ is a function of x which satisfies equation (i).*

* The above proposition is true for any linear differential equation whatever; that is when the coefficients A, B, etc., are constants, and when the coefficients A, B, etc., are functions of the independent variable x. The latter type of linear differential equation does not, however, concern us here.

The above proposition is the basis of Fourier's method of analysis as applied to the flow of heat and as applied to the motion of strings, and it is the basis of the use of spherical, zonal, and cylindrical harmonics. The importance of the proposition can scarcely be overestimated.

Undetermined constants in the solution of an ordinary differential equation.—It is sufficient, perhaps, to illustrate this matter by two very simple examples.

Example 1.—Consider the simple ordinary differential equation

$$\frac{dy}{dt} = a$$

where a is a given constant. The increase of y during t seconds is at, and the value of y at the end of the t seconds is

$$y = at + C$$

where C is the unknown value of y at the beginning $(t = 0)$.

Example 2.—Consider the simple ordinary differential equation

$$\frac{d^2y}{dt^2} = a$$

where a is a given constant. Then

$$\frac{dy}{dt} = at + B$$

and

$$y = \tfrac{1}{2}at^2 + Bt + C$$

where C is the unknown value of y at the beginning, and B is the unknown value of dy/dt at the beginning.

Note.—From the point of view of the physicist the unknown constants which appear in the general solution of an ordinary differential equation are thought of as *disposable constants* because the solution may be made to fit any special case by assigning proper values to these constants. The number of disposable

constants in the general solution of an ordinary differential equation is always equal to the order of the differential equation.

Undetermined functions in the solution of a partial differential equation.—It is sufficient, perhaps, to illustrate this matter by a few simple examples.

Example 1.—Concerning a hill it is known only that its slope, $\partial y/\partial x$, in the direction of the x-axis is constant and equal to a so that

$$\frac{\partial y}{\partial x} = a \tag{i}$$

Before the complete hill or surface can be constructed from this differential equation an arbitrary starting curve cc, Fig. 74, must be chosen. Let $y = F(z)$ be the equation to the curve cc, then $F(z)$ is the height of the hill at any point z on cc, and $ax + F(z)$ is the height of the hill at any point x, z. That is to say, the integration of (i) gives

$$y = ax + F(z) \tag{ii}$$

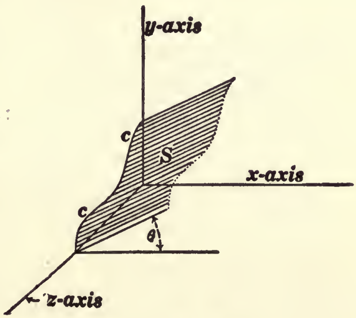

Fig. 74.

Example 2.—If all boys were of the same ability we might say that any boy saves money at the average rate of \$5 per year beginning at fourteen years of age. Integrating with respect to b, the boy, from fourteen years to twenty-one years we get \$35; but the amount of money a boy has when he comes of age is not \$35 plus a *constant*, but \$35 + $F(m)$, where $F(m)$ is what the boy's "old man" has saved for him; b and m are independent variables, let us say, and a "constant" of integration with respect to b turns out to be an unknown function of m.

Example 3.—Concerning a hill it is known only that its slope, $\partial y/\partial x$, in the direction of the x-axis increases a times as fast

as x, or, expressed in symbols, we have

$$\frac{\partial^2 y}{\partial x^2} = a \tag{iii}$$

Before the complete hill or surface can be constructed from this differential equation two things must be given, namely, (1) An arbitrary starting curve like cc, Fig. 74, and (2) An arbitrary value of the starting slope, $\partial y/\partial x$, at each point of cc.

The integration of (i) or (iii) is called *partial integration*, but example 2 should make it clear that partial integration is identical to ordinary integration, the only difference being that in the former the "constant" of integration turns out to be a function of the other independent variable or variables. Therefore, integrating (iii) twice we get

$$y = \tfrac{1}{2}ax^2 + x \cdot f(z) + F(z) \tag{iv}$$

where $f(z)$ and $F(z)$ are unknown functions of z. Indeed $y = F(z)$ is the equation to the starting curve cc in Fig. 74, and the value of the starting slope $(\partial y/\partial x)_{x=0}$, at each point of cc is $(\partial y/\partial x)_{x=0}, = f(z)$.

Note.—From the point of view of the physicist the unknown functions which appear in the general solution of a partial differential equation are thought of as *disposable functions* because the solution may be made to fit any special case by properly choosing the forms of these functions. The number of these disposable functions in the general solution of a partial differential equation is always equal to the order of the differential equation.

General solution of equation (3).—Equations (1) and (2) are particular solutions of (3), and therefore

$$y = F(x - vt) + f(x + vt) \tag{4}$$

is also a solution, according to the principle of superposition. But equation (4) involves two independent and disposable functions and it is therefore the general solution of (3).

Equation (4) represents the piling on top of each other of two curves of any shape, one of the curves traveling to the right at velocity v, and the other traveling to the left at velocity v.

Differential equation of motion of a stretched string.—When a stretched string is in equilibrium it is of course straight. Let us choose this equilibrium position of the string as the x-axis of reference. We will assume that each particle of the string moves only in a direction at right angles to the string (parallel to the y-axis of reference), and we will assume that the string is perfectly flexible which means that the only forces to be considered are the forces due to the tension of the string. An important consequence of the first assumption is that the x-component of the tension of the string has always and everywhere a certain value T which is equal to the tension of the string when it is in equilibrium.

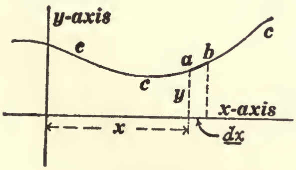

Fig. 75.

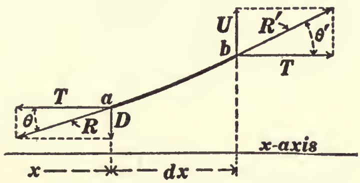

Fig. 76.

Let the curve ccc, Fig. 75, be the configuration of the string at a certain instant, that is, ccc is what the photographer would call a snapshot of the moving string. The shape of the curve ccc defines y as a function of x and the steepness of the curve at any point is the value of $\partial y/\partial x$* at that point.

Consider the very short portion ab of the string. The length of this portion when the string lies along the x-axis (in equilibrium) is dx, and the mass of the portion is $m \cdot dx$, where m is the mass per unit length of string. An enlarged view of the very short portion ab of the string is shown in Fig. 76. The

* Any reader who fails to appreciate the fact that we are here talking about the state of affairs at a given instant, or that time is supposed to stop, as it were, may wonder why we use the notation $\partial y/\partial x$ instead of dy/dx.

adjacent portions of the string pull on the portion ab, the pull at a is represented by R and it is parallel to the string at a, and the pull at b is represented by R' and it is parallel to the string at b. The x-component of R is the force T to the left, and the x-component of R' is an equal force T towards the right. Therefore the downward force D (see Fig. 76) is equal to $T \tan \theta$, the upward force U is equal to $T \tan \theta'$, and the net upward force acting on the portion ab of the string is:

$$dF = U - D = T \tan \theta' - T \tan \theta \qquad \text{(i)}$$

But $\tan \theta$ is equal to the value of $\partial y/\partial x$ at a, and $\tan \theta'$ is equal to the value of $\partial y/\partial x$ at b. Therefore the value of $\tan \theta' - \tan \theta$ is the increase of $\partial y/\partial x$ from a to b, and this increase is equal to $(\partial^2 y/\partial x^2) \cdot dx$. This is evident when we consider that $\partial^2 y/\partial x^2$ means the rate of increase of $\partial y/\partial x$ with respect to x. Therefore, substituting $(\partial^2 y/\partial x^2) \cdot dx$ for $\tan \theta' - \tan \theta$ in equation (i) we get:

$$dF = T \frac{\partial^2 y}{\partial x^2} \cdot dx \qquad \text{(ii)}$$

Now, according to Newton's laws of motion, the net upward force dF acting on the portion ab of the string is equal to the mass $m \cdot dx$ of the portion multiplied by the upward acceleration, $\partial^2 y/\partial t^2$, of the portion. Therefore substituting $m(\partial^2 y/\partial t^2) \cdot dx$ for dF in equation (ii) we get:

$$m \frac{\partial^2 y}{\partial t^2} = T \frac{\partial^2 y}{\partial x^2}$$

or

$$\frac{\partial^2 y}{\partial t^2} = \frac{T}{m} \cdot \frac{\partial^2 y}{\partial x^2} \qquad \text{(3) } bis$$

The general solution of this differential equation, as explained above, is:

$$y = F(x - vt) + f(x + vt) \qquad \text{(4) } bis$$

where v is given by the equation:

$$v = \sqrt{\frac{T}{m}} \tag{5}$$

Of course particular solutions of equation (3) are

$$y = F(x - vt) \tag{6}$$

and

$$y = f(x + vt) \tag{7}$$

Equation (6) represents a bend or curve on the string traveling to the right at velocity v and retaining its shape unchanged, and equation (7) represents a bend or curve on the string traveling to the left at velocity v and retaining its shape unchanged. These traveling bends of unchanging shape are called *pure waves*.

Pure wave traveling to the right.—Equation (6) expresses what is called a *pure wave* traveling to the right and it is important to consider the necessary relation between V and s where V is the sidewise velocity and s is the slope of the string at a point; of course, $V = \partial y/\partial t$ and $s = \partial y/\partial x$.

From equation (6) we have $s = (\partial y/\partial x) = F'(x - vt)$, and $V = \partial y/\partial t = - v \cdot F'(x - vt)$, and therefore for a pure wave traveling to the right we have

$$\frac{V}{s} = - v \tag{8}$$

Pure wave traveling to the left.—From equation (7) we have $s = \partial y/\partial x = F'(x + vt)$ and $V = \partial y/\partial t = + v \cdot F'(x + v)$, and therefore for a pure wave traveling to the left we have

$$\frac{V}{s} = + v \tag{9}$$

Remark.—Equations (8) and (9) are useful in that they enable one to pick out particular solutions of equation (3). Any distribution of sidewise velocity V and slope s which satisfies (8) is a pure wave traveling to the right, and any distribution of

sidewise velocity V and slope s which satisfies (9) is a pure wave traveling to the left.

Reflection and change of phase thereby.—Let us consider a simple form of pure wave traveling to the right along a string, a wave throughout which the sidewise velocity of the string has everywhere the same value V, and throughout which the slope of the string has everywhere the same value s. Such a simple wave we will call a *ribbon wave*, and it may be symbolized by the arrow in Fig. 77. The head of the arrow shows the direction of travel, everywhere between the points a and b the string is moving sidewise at velocity V, and everywhere between a and b the slope of the string is s. This slope is not actually shown however.

What happens when the wave $V s$ reaches the end of the string? Whatever happens on the string it must be of the nature of a pure wave traveling to the right and a pure wave traveling to the left, one heaped on top of the other, because this is the physical interpretation of the general equation (4). Therefore if we assume a pure wave (sidewise velocity V' and slope s') to

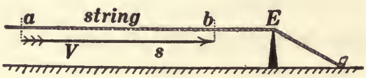

Fig. 77.

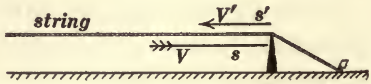

Fig. 78.
Reflection from rigid end of string.

shoot to the left from E when ab reaches E we are sure to cover every possible outcome, and then if this assumption is justified by showing that it satisfies all of the necessary conditions we may be sure that the correct solution has been found. In Fig. 78, therefore, is shown the tail end of the original wave $V s$, and the beginning of an assumed reflected wave $V' s'$. Now, according to equations (8) and (9) we must have

$$\frac{V}{s} = -v \qquad \text{(i)}$$

and

$$\frac{V'}{s'} = +v \qquad \text{(ii)}$$

Furthermore the actual end of the string cannot move and therefore we must have

$$V + V' = 0 \qquad \text{(iii)}$$

and from equations (i), (ii) and (iii) it follows that $V' = -V$ and that $s' = s$. That is to say, a wave is wholly reflected from the rigid end of a string (numerical values of V' and V the same, and numerical values of s and s' the same), and the sidewise velocity of the string in the reflected wave is the reverse of the sidewise velocity of the string in the original wave ($V' = -V$).

Figure 79 represents an ideal condition in which the end of a string is held by an indefinitely long weightless thread. In this case the actual end of the string cannot slope, and therefore we must have

$$s + s' = 0 \qquad \text{(iv)}$$

which in conjunction with equations (i) and (ii) gives $s' = -s$

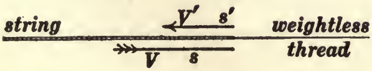

Fig. 79.
Reflection from "free" end of string.

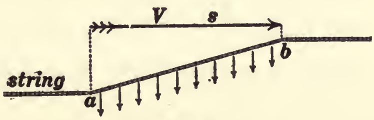

Fig. 80.

and $V' = V$. That is, a wave is wholly reflected (numerical values of s and s' the same, and numerical values of V and V' the same) at the "free" end of a stretched string, and the slope in the reflected wave is opposite to the slope in the original wave ($s' = -s$). The "free" end of a stretched string as shown in Fig. 79 is merely an ideal, but the analogous condition in electric waves and in air waves is very common.

Examples of wave motion on a string.—1. *A complete picture of a ribbon wave* traveling to the right on a stretched string is shown in Fig. 80. The region ab occupied by the wave has a uniform slope s, and every part of the string ab is moving downwards at velocity V as indicated by the small parallel arrows.

2. *An indefinitely long stretched string is struck sharply by a square-faced hammer* so as to set every particle of the portion ab of the string moving sidewise at a given velocity V as indicated by the small parallel arrows in Fig. 81. To determine the motion of the string it is only necessary to resolve the uniform sidewise

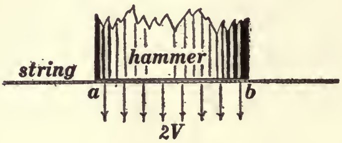

Fig. 81.

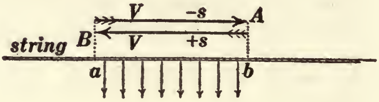

Fig. 82.

velocity and zero slope of portion ab into two oppositely traveling pure waves as indicated in Fig. 82, in which the arrow A represents a ribbon wave traveling to the right and the arrow B represents a ribbon wave traveling to the left. Half of the given sidewise velocity is associated with each ribbon wave as indicated, and equal and opposite slopes [according to equations (8) and (9)] are associated with the respective ribbon waves. The state of affairs a moment later is shown in Fig. 83, and Fig. 84 shows the

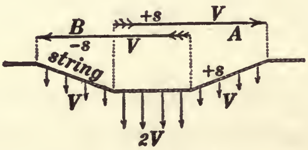

Fig. 83.

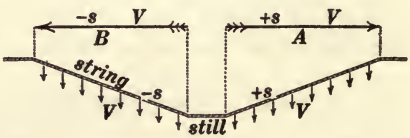

Fig. 84.

state of affairs after the two ribbon waves A and B have entirely separated from each other.

3. *Motion of a plucked string.*—A stretched string is pulled sidewise into the position APB, Fig. 85, and released; and it is required to determine the motion of the string. For the sake of simplicity the point P (or C) is taken at the middle of the string.

To determine the motion of the string the initial condition of the string, namely, slope $+a$ between A and C, and

slope $-a$ between C and B, with no sidewise velocity anywhere, must be resolved into pure waves traveling to right and left. Thus the slope $-a$ is resolved into the two waves $s'\ V'$ and $s''\ V''$, and the slope $+a$ is resolved into the two waves $s'''\ V'''$ and $s^{IV}\ V^{IV}$.

Now V' must be equal to $-V''$ because there is zero sidewise velocity between C and B, therefore, according to equa-

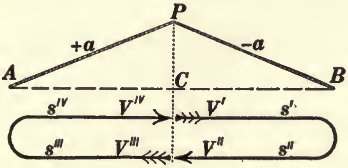

Fig. 85.

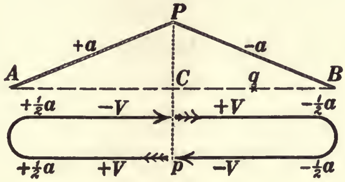

Fig. 86.

tions (8) and (9), s' must be equal to s'' and the same in sign. But $s' + s'' = -a$ so that $s' = s'' = -a/2$.

Similarly, V''' must be equal to $-V^{IV}$, so that s''' must be equal to S^{IV} and the same in sign. But $S''' + S^{IV} = +a$ so that $S''' = S^{IV} = +a/2$.

Therefore, using $a/2$ for the common numerical value of s', s'', s''' and s^{IV}, and using V for the common numerical value of V', V'', V''' and V^{IV}, and indicating the proper sign in each case we may simplify Fig. 85 as indicated in Fig. 86.

To find the state of affairs after lapse of time t it is only necessary to consider that the ribbon waves will all have traveled forwards a distance vt, and that reflection at A and B always takes place with reversal of V. Thus after time sufficient for waves to travel over $1\frac{1}{4}$ times the length of the string (arrowhead p, for example, will have traveled to A and back to q,

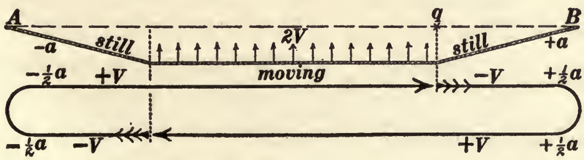

Fig. 87

and $-V$ will have been converted into $+V$ by reflection at A) the state of affairs will be as shown in Fig. 87. A portion of the string near A is still and its slope is $-a$; the middle portion of the string is moving upwards at velocity $2V$ and its slope is zero; and the portion near B is still and its slope is $+a$. Slope and velocity of each part of string in Fig. 87 is found by adding slopes and velocities associated with the overlapping portions of the respective ribbon waves.*

The Kelvin ladder.—A number of equidistant bars are fixed to a fine steel wire or ribbon and suspended from a small crank as indicated in Fig. 88. The rotatory motion of each portion of this arrangement satisfies the equation

$$\frac{\partial^2\phi}{\partial t^2} = \frac{E}{k} \cdot \frac{\partial^2\phi}{\partial x^2}$$

where ϕ is the angular displacement at the instant t of the bar which is at a distance x below the upper end of the ladder (x-axis directed downwards), k is the moment inertia of unit length of the ladder, and E is the torque required to produce one radian of twist per unit length of the ladder. This equation may be established by an argument which is nearly identical in form to the argument leading to equation (3) on page 176, and what we may call a ribbon wave may be symbolized by the arrow in Fig. 88. Every portion of the ladder between the points a and b is rotating at uniform spin-velocity, and the whole of the portion ab of the ladder is uniformly twisted. Let ω be the spin-velocity and h the degree of twist (radians per unit length of ladder). Then the ratio ω/h has a certain value, a, which is positive or negative according to whether the wave is traveling upwards or downwards.

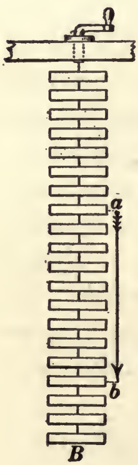

Fig. 88.

* A more elaborate example of the motion of a plucked string as analyzed by this same method is given in *Journal of the Franklin Institute*, Vol. CLXXIX, May, 1915.

When a ribbon wave is reflected at the free end B of the ladder, the twist h is reversed without change of sign of ω. That is to say, if the ladder is twisted like a right-handed screw or helix in the original wave it will be twisted as a left-handed screw or helix in the reflected wave, but the rotation or spin in the reflected wave will be in the same direction as the rotation or spin in the original wave.

When the end bar at B is rigidly fixed, a ribbon wave is reflected at B with reversal of rotation or spin but without reversal of twist.

What takes place when an elastic rod strikes endwise against a rigid wall and rebounds.—A longitudinal wave on a rod is a state of endwise compression C (negative value of C means tension) associated with a certain velocity V of the material of the rod, and the ratio V/C has a certain value, a, which is positive or negative according to the direction of travel of the wave. The initial condition of the elastic rod just before it strikes the wall is a uniform velocity of the material of the rod towards the wall, this initial condition is to be thought of as always being in existence except in so far as it is modified by the superposition of new conditions, and this initial condition of uniform unchanging motion is symbolized by the heavy dotted line in Figs. 89 and 90.

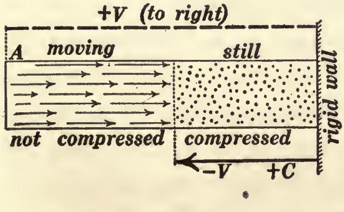

Fig. 89.

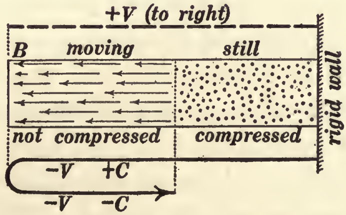

Fig. 90.

After the rod strikes the wall it continues for a definite time to push steadily against the wall, a long-drawn-out ribbon wave continues to shoot out from the wall as indicated by the heavy-

line arrow in Figs. 89 and 90. The first lap of this ribbon wave (velocity $-V$ of the material of the rod and an associated compression C) annuls or literally wipes out the initial velocity V of the rod, and lays down a condition of uniform compression as indicated in Fig. 89. The second lap of the ribbon wave (velocity $-V$ and compression $-C$, because reflection at the free end B of the rod takes place with reversal of C so that the compression in the first lap becomes tension in the second lap) then wipes out the uniform compression and lays down a uniform condition of reversed motion as indicated in Fig. 90. Therefore when the second lap of the ribbon wave reaches the wall the entire rod is relieved of compression and it is moving away from the wall at velocity V.

Sudden stopping of flow of water in a rigid pipe.—When a valve is suddenly closed at the end of a pipe the stoppage and rebounding of the column of moving water takes place precisely in the manner of the stoppage and rebounding of the rod as represented in Figs. 89 and 90, on the assumption that the pipe is rigid.

Experiment with a cast iron rod.—A short slug moving at velocity $2V$ comes squarely against the end of a rod as indicated by diagram A in Fig. 91. Let us assume that slug and rod are of the same diameter and made of the same material. Then as long as the slug pushes on the rod it may be thought of as a part of the rod, and the state of affairs as shown in diagram A is resolvable into two oppositely moving waves (ribbon waves) $V - C$ and $V + C$ as shown. The wave $V - C$ merges into $V + C$ as it is reflected from the free end of the slug, and soon a ribbon wave of length $2L$ (where L is the length of the slug) develops and travels to the right along the rod as indicated in diagrams B and C. This ribbon wave is reflected from the right-hand end of the rod as indicated in diagram D, a region of doubled velocity $2V$ and zero compression develops until the ribbon wave is half reflected as shown in diagram E, and then a region R of tension begins to develop as indicated in

diagram *F*. This tension is equal to the original compression, and, if the rod is made of cast iron which withstands very great compression but which does not withstand great tension it may easily be that the rod separates in the narrow region *R* in dia-

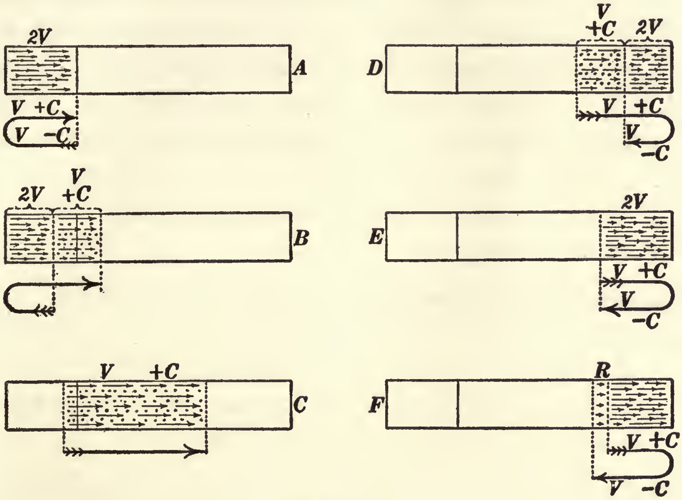

Fig. 91.

gram *F*. The imperfect elasticity and lack of homogeneity of a cast-iron rod modifies the ideal action as represented in Fig. 91, and some appreciable time is required for the cast-iron rod to separate in the region *R*. Therefore the slug which flies off the end of the cast-iron rod is sure to be somewhat longer than the original slug in diagram *A*, and it will be moving as a whole at a lower velocity than $2V$. Nevertheless it is a very interesting and instructive experiment to shoot a steel slug against the end of a cast-iron rod.

Use a No. 10 single-barrel shot gun; use a snugly fitting steel slug about 2 inches long; make the cast-iron rod of same diameter as steel slug with a squarely ground end; and use about half-a-gram of black powder with an empty space of two inches or more between powder and slug. This experiment is suggestively frightful but in fact quite safe.

An interesting example of the action which is represented in Fig. 91 is afforded by some experiments which were made at Woolwich, England, to determine the effect of exploding a charge of dynamite, or other high explosive, against a thick plate of steel armor. In some of the experiments a high bulge was formed on the back of the plate, and when the plate was cut open its section was as shown in Fig. 92. A deep dent was formed where the explosion took place, a layer of steel t was thrown off the other side of the plate, and a hollow space h was left.

Fig. 92.

Experiment with billiard balls.—Place a number of billiard balls together in a straight row. Let one ball, two balls, three balls, etc., come against the end of the row, and one ball, two balls, three balls, etc., will fly off from the other end of the row. The action is essentially the same as that shown in Fig. 91.

Whip action.—An interesting modification of the above described experiment of shooting a steel slug against the end of a cast-iron rod is to use a tapering cast-iron rod and shoot the steel slug against the larger end of the rod. As the ribbon wave travels towards the smaller end of the rod the values of V and C increase, and a much lower initial velocity of slug is sufficient to throw off the tip of the tapered rod.

This wave intensification along a tapered rod is sometimes troublesome in modern high-power guns which taper from breech to muzzle. A quick-acting backward force on the breech block produces a ribbon wave of tension-and-backward-motion, and as this ribbon wave travels towards the muzzle it is intensified, and the tension may rise to a value exceeding the strength of the material, thus breaking off the muzzle of the gun, or, what is quite as bad for the gun, leaving a permanent stretch of the muzzle portions of the gun.

The intensification of a wave which travels along a tapered

rod is an example of what Professor P. G. Tait called *whip action*, and Professor Tait's explanation of the cracking of a whip is as follows: First, any object moving through the air at a velocity greater than the velocity of sound produces a sharp snap or crack. Second, a wave of moderate sidewise velocity may be started in the thick and heavy butt portions of a whip, and the sidewise velocity increases more and more as the wave travels towards the small end of the whip, thus causing the whip cracker to reach velocities exceeding 1,000 feet per second.

An interesting experiment is to stand on a ladder, hold the butt end of a long flexible whip in the hand, let the whip hang vertically downwards, move the butt quickly to one side, and watch the increasing visible amplitude of motion of the resultant wave as it travels down the whip.*

Gun recoil. Simple ideal case.—A portion of the recoil of a heavy gun (backward momentum of gun, the gun being assumed to be free) is equal to the forward momentum of the projectile, and the remainder is equal to the forward momentum of the powder gases. The first part of the recoil momentum is quite easily calculated whereas no rational method has hitherto been developed for calculating the second part of the recoil momentum.† A very simple and more or less ideal example of gas-recoil would be afforded by suddenly opening one end of a tube which contains very slightly compressed air. The initial condition of uniform compression is symbolized by the heavy dotted line in Figs. 93 and 94, and the heavy arrow symbolizes the long-drawn-out ribbon wave (of outward-motion-and-rarefaction, $+V$ and $-C$) which continues to shoot inwards from the open end of the tube. The first lap of this ribbon wave wipes out the existing compression and lays down a condition of out-

* Change of wave velocity due to changing mass and tension have an important effect here, but it is not worth while to discuss the matter further.

† This deficiency has recently been met in a paper by Wm. S. Franklin, *Journal of the Franklin Institute*, Vol. CLXXIX, pages 559–577, May, 1915.

ward motion. The ribbon wave is reflected from the closed end of the tube with reversal of V, and the second lap of the ribbon wave wipes out the outward motion and lays down a condition of rarefaction. The ribbon wave is then reflected

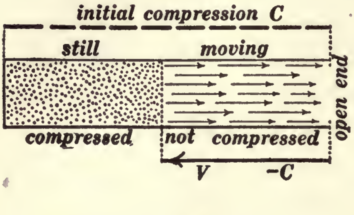

Fig. 93.

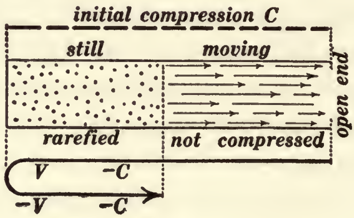

Fig. 94.

from the open end with reversal of C, and the third lap of the ribbon wave wipes out the rarefaction and lays down a condition of inward motion. The ribbon wave is then reflected from the closed end of the tube with reversal of V and the fourth lap of the ribbon wave wipes out the inward motion and lays down a condition of uniform compression as at the beginning.

If such a simple wave motion existed in the powder gases of a gun, the gas-recoil momentum would be due to the fact that the gases would continue to push backwards on the breech block after the opening of the muzzle for the length of time required for the first lap of the ribbon wave to reach the breech block. As a matter of fact, however, the wave motion in the powder gases is greatly complicated by the velocity already existent in the gases when the muzzle is opened and by the adiabatic cooling of the powder gases as they expand.

Experiment with Kelvin ladder showing the effect described in connection with Figs. 93 and 94.—Clamp the crank at the top of the ladder, take hold of the bar at the bottom and twist the ladder slightly, and wait for the ladder to become still. Then release the bottom bar. The initial state of twist is analogous to the initial compression of the air in a tube, and the initial twist may be thought of as symbolized by the heavy dotted line in

Figs. 93 and 94. Releasing the bottom bar is analogous to the opening of the end of the tube. A ribbon wave (of rotation-and-reversed-twist) then wipes out the existing twist and lays down a condition of uniform rotation. At the instant this ribbon wave reaches the top of the ladder the ladder will be seen to be *flat* (entirely freed from twist) and the entire ladder will be in uniform rotation. The further details of behavior of ladder need not be described but they should be fully described if the experiment is shown to a class, otherwise the student will not know what to look for.

Differential equations of electrical wave motion on a transmission line.—The theory of electrical wave motion is usually developed in terms of electric and magnetic field intensities in space,* and the equations are not easy to understand, especially by the electrical engineer who is accustomed to express everything in terms of voltage and current. Therefore the following discussion of electric wave motion on a transmission line is expressed in terms of voltage and current. Throughout the discussion the resistance of the line wires is assumed to be negligible and the line wires are assumed to be perfectly insulated.

The horizontal lines in Fig. 95 represent the wires of a trans-

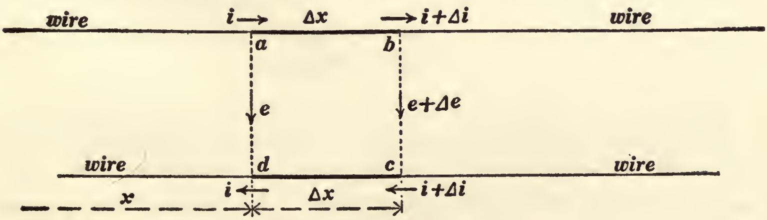

Fig. 95.

mission line, and *abcd* is an *element* of the line. Let e be the voltage across the transmission line at the point *ad* and let i

* A very simple development of this general theory is given on pages 186–195 of Franklin's *Electric Waves*, The Macmillan Co., 1909. The student must be familiar with the elements of vector analysis to be able to understand this electromagnetic theory. See *Electric Waves*, pages 158–185, or see Franklin, MacNutt and Charles's *Calculus*, pages 210–253.

be "the current in the line at the same point" (meaning outflowing current in one wire and returning current in the other wire) as shown in Fig. 95. The voltage across the line at the point bc is $e + \Delta e$, and the current in the line at the same point is $i + \Delta i$.

The capacity of the element $abcd$ is $C \cdot \Delta x$, where C is the *capacity of unit length of the line.* Therefore the charge q "on the element" (positive charge on ab and negative charge on cd) is $q = C \cdot \Delta x \times e$,* and the rate of decrease of q, namely, $-\frac{\partial e}{\partial t} \cdot C \cdot \Delta x$, is equal to Δi. Therefore we have:

$$C \frac{\partial e}{dt} = - \frac{\partial i}{\partial x} \qquad \text{(i)}$$

The net electromotive force around the elementary circuit $abcd$ is $(e + \Delta e) - e$, and this electromotive force causes the current in the circuit to decrease† at a definite rate such that $\Delta e = - L \cdot \Delta x \times \frac{\partial i}{\partial t}$, according to Art. 196, pages 276–277, *General Physics*, where L is the *inductance per unit length* of the transmission line and $L \cdot \Delta x$ is the inductance of the elementary circuit $abcd$. Therefore we have

$$L \frac{\partial i}{\partial t} = - \frac{\partial e}{\partial x} \qquad \text{(ii)}$$

Equations (i) and (ii) contain the two unknown dependent variables e and i, and it is necessary to eliminate one to get an equation involving the other alone. By differentiating equation (i) with respect to t and equation (ii) with respect to x

* The charge is greater than $C \cdot \Delta x \times e$ and less than $C \cdot \Delta x \times (e + \Delta e)$, and when Δx approaches zero, the expression for q approaches $C \cdot \Delta x \times e$ as a limit.

† The electromotive force $e + \Delta e$ is associated with an electric field from wire to wire and the arrow shows the direction of this field or the direction of $e + \Delta e$ as it would be indicated by a voltmeter. Evidently, however, an excessive charge on the wires at bc and a large electromotive force from b to c would tend to create a current opposite to i.

we get

$$C\frac{\partial^2 e}{\partial t^2} = -\frac{\partial^2 i}{\partial x\cdot\partial t} \quad \text{and} \quad L\frac{\partial^2 i}{\partial t\cdot\partial x} = -\frac{\partial^2 e}{\partial x^2}$$

but $\partial^2 i/(\partial x\cdot\partial t) = \partial^2 i/(\partial t\cdot\partial x)$,* and therefore we get

$$\frac{\partial^2 e}{\partial t^2} = \frac{1}{LC}\cdot\frac{\partial^2 e}{\partial x^2} \tag{10}$$

By differentiating equation (ii) with respect to t and equation (i) with respect to x, and eliminating as before, we get

$$\frac{\partial^2 i}{\partial t^2} = \frac{1}{LC}\cdot\frac{\partial^2 i}{\partial x^2} \tag{11}$$

The general solution of equation (10) is of course

$$e = F(x - vt) + f(x + vt)$$

where

$$v = \sqrt{\frac{1}{LC}} \tag{12}$$

but it is most convenient for present purposes to consider the two particular solutions $e = F(x - vt)$ and $e = f(x + vt)$ as follows:

A pure wave traveling to the right in Fig. 95 is represented by the particular solution

$$e = F(x - vt) \tag{13}$$

and to adopt this solution of (10) is to fix the corresponding solution of (11) as follows: From equations (i) and (13) we have

$$-Cv\cdot F'(x - vt) = -\frac{\partial i}{\partial x} \tag{iii}$$

and from equations (ii) and (13) we have

$$L\frac{\partial i}{\partial t} = -F'(x - vt) \tag{iv}$$

* This is by no means self-evident as an arithmetical proposition; but a full discussion of it would be out of place here.

Therefore, by integrating (iii) and (iv), we get

$$i = Cv \cdot F(x - vt) + \text{any function of } t \tag{v}$$

and

$$i = \frac{1}{Lv} \cdot F(x - vt) + \text{any function of } x \tag{vi}$$

But, according to equation (12), $Cv = \frac{1}{Lv} = +\sqrt{\frac{C}{L}}$, and therefore the unknown function of t in (v) must be identical to the unknown function of x in (vi), which means, in the most general case, that both functions are constants; and for present purposes these constants may be taken as zero. Therefore equations (v) and (vi) become

$$i = a \cdot F(x - vt) \tag{14}$$

where

$$a = +\sqrt{\frac{C}{L}} \tag{15}$$

whence dividing (14) by (13) we get

$$\frac{i}{e} = +a \tag{16}$$

From equations (13), (14) and (15) it is evident that a pure wave traveling to the right in Fig. 95 is *any distribution of voltage* e *traveling to the right at velocity* v, *the current at each point in the line being equal to* ae; or any distribution of current i traveling to the right at velocity v, the voltage across the line at each point being equal to i/a.

A pure wave traveling to the left in Fig. 95.—Starting with $e = f(x + vt)$ we may find, by an argument similar to the above, that $i = -ae$. That is

$$\frac{i}{e} = -a \tag{17}$$

and it follows that a pure wave traveling to the left in Fig. 95 is *any distribution of voltage* e *traveling to the left at velocity* v, *the current at each point in the line being equal to* $-ae$.

Remark.—Equations (16) and (17) are useful in that they enable one to pick out particular solutions of equations (10) and (11).

Above discussion from another point of view.—Professor P. G. Tait's classical discussion of wave motion on a stretched string which does away with all necessity for the solution of differential equations* has its analog in the theory of electrical waves, and it is interesting to note that the avoidance of integration comes from the postulate of travel which is introduced at the start. This fact makes it all the more clear that the classical differential equation of wave motion [see equation (10) or (11) above] is merely a differential equation of travel.

Effect of traveling distribution of current.—Imagine current to be distributed over a transmission line in any arbitrary manner, the current i at any given point ad of the line (meaning outflowing current i in one wire and back flowing current i in the other wire) being represented by the ordinate i of any given curve CC, Fig. 96, and suppose this current distribution to travel as a whole to the right in Fig. 96 at velocity v, that is to say, let us imagine the curve CC to travel to the right at velocity v and the current distribution to change so as to be represented at each instant by this traveling curve. It is strictly meaningless to think of the current itself as moving along, but it is convenient to think of the current and the magnetic field (also the charges on the wires and the electric field between the wires) as moving along with CC. Such a traveling current distribution would produce a voltage distribution (traveling along with it) such that

$$e = Liv \qquad (18)$$

where e is the voltage across the line at the place where the current in the line is i. This equation may be established as follows: The inductance of the element $abcd$ is $L \cdot \Delta x$, and the magnetic flux between the wires ad and bc is equal to the

* See *General Physics*, Art. 359, pages 499–501.

inductance of the element multiplied by the current, everything being expressed in c.g.s. units. Now, the current distribution and the associated flux are assumed to travel to the right at velocity v so that all of the flux between ad and bc will sweep across the line bc in $\Delta x/v$ of a second. Therefore the voltage e induced along bc by the traveling flux is $Li \cdot \Delta x$ divided by $\Delta x/v$ which gives Liv abvolts.

Effect of traveling distribution of voltage.—Imagine electric charge to be distributed over the transmission line in Fig. 96

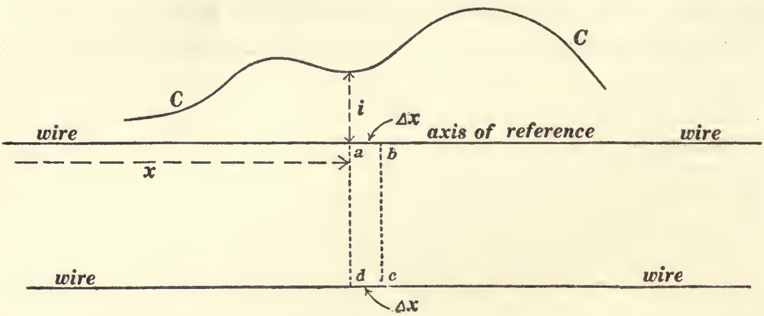

Fig. 96.

(positive charge on one wire and an equal negative charge on the other). This charge means a definite voltage between wires at each point along the line, and we may imagine the *voltage* e at each point to be represented by the ordinate of a curve CC. Imagine the electric charge and the associated voltage distribution to travel along the line at velocity v. Such a traveling voltage distribution would produce a definite current distribution over the line such that

$$i = Cev \tag{19}$$

where i is the current in the line at the place where the voltage across the line is e, and C is the capacity of the line per unit of length. This equation may be established as follows: The capacity of the element $ab\ cd$ is $C \cdot \Delta x$ so that $C \cdot \Delta x \times e$ is the positive charge on ab (or negative charge on cd). But all of the charge on ab must flow past the point b during

$\Delta x/v$ of a second, so that the current in the line is found by dividing $C \cdot \Delta x \times e$ by $\Delta x/v$ which gives equation (19). Of course the current distribution travels along with voltage distribution which produces it.

Mutually sustaining current and voltage distributions.—Let e, i and v refer to identically the same quantities in equations (18) and (19) respectively, that is to say, e is produced by the motion of i, and in turn i is produced by the motion of e (see pages 100–105 of this volume). Then equations (18) and (19) are simultaneous equations, and by elimination we get

$$v = \sqrt{\frac{1}{LC}} \tag{20}$$

and

$$\frac{i}{e} = \pm \sqrt{\frac{C}{L}} \tag{21}$$

A clear understanding of an electric wave traveling along a transmission line may be obtained from Fig. 97, resistance of

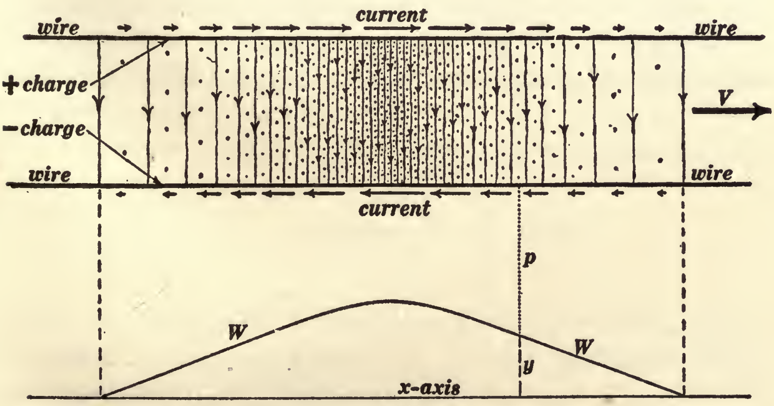

Fig. 97.

Showing an electromagnetic wave on a transmission line.

line wires being negligible and wires being perfectly insulated. The curve WW corresponds to CC in Fig. 96, and the ordinate y represents the voltage e across the line or the current i

in the line at the point p. The upper wire is positively charged and the fine vertical lines represent the lines of force of the electric field which emanates from the positively charged wire and converges upon the lower wire which is negatively charged. Current flows to the right in the upper wire and to the left in the lower wire as represented by the short horizontal arrows, and the fine dots represent the lines of force of the magnetic field between the wires, this magnetic field being perpendicular to the plane of the paper in Fig. 97. The heavy arrow shows the direction of travel of the wave at velocity v.

The direction of travel and the directions of e and i may be correlated as follows: That particular wire is positively charged out of which the electric lines of force emanate; the voltage e is from positively charged wire to negatively charged wire; and *the current in the positively charged wire may be thought of as carrying the positive charge forwards from the back of the wave and laying it down on the wire in front of the wave.*

What determines wave form?—A device which produces a varying voltage e and which can deliver current *ad libitum* is connected across the end of a transmission line, and the curve in Fig. 99 (which is of course traveling to the right) which represents the shape of the wave which shoots out along the transmission line is exactly like the curve in Fig. 98 which shows e as a

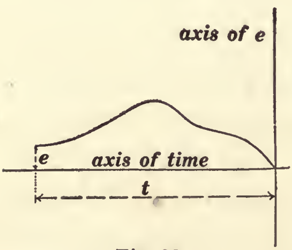

Fig. 98.

Showing voltage e as a function of the time.

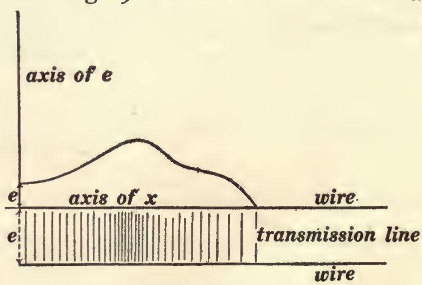

Fig. 99.

Showing shape of wave which shoots out from end of a transmission line when voltage e is connected across end of line. The wave involves a certain current $i = ae$ at each part of line according to equation (16).

function of the time. This is evident when we consider, first, that the assumed wave satisfies the differential equations (10) and (11), second, that the assumed wave is consistent with the fact that the transmission line is entirely quiescent up to the instant of connecting e, and third, that the assumed wave in passing out on the line in Fig. 99 involves at each instant a voltage e across the end of the line which is in fact equal to the voltage acting at that instant according to Fig. 98. That is to say, first, the differential equations are satisfied, second, the initial conditions are satisfied, and third, the boundary conditions are satisfied.

The wave train.—When a periodic electromotive force acts across the end of the line, for example when an alternator is connected to the line, then what is called a *wave-train* passes out along the line, and the state of affairs (before matters are complicated by the reflection of the waves from the distant end of the line) is shown in Fig. 100. This figure shows a wave-train

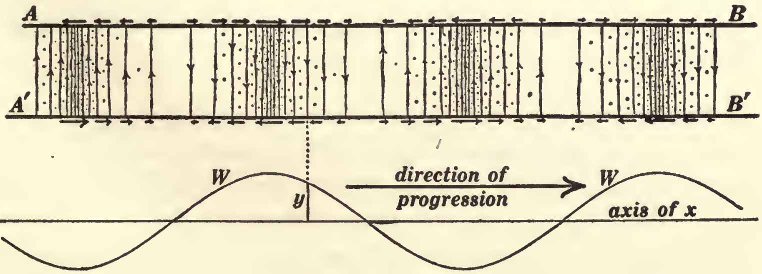

Fig. 100.

which is produced by a harmonic alternating voltage, and the curve WW is a curve of sines. The short horizontal arrows represent the current at various places in the wires, the fine vertical lines represent the lines of force of the electric field, and the dots represent the lines of force of the magnetic field as in Fig. 97.

The ribbon wave.—When a battery of constant voltage and negligible resistance is connected across the end of a transmission

line, a wave shoots out on the line, the voltage e in the wave is everywhere of the same value and equal to battery voltage, and the current is everywhere the same in value and equal to $e \times \sqrt{C/L}$, according to equation (16). Such a wave we will call a *ribbon wave.* Thus Fig. 101 represents a ribbon wave d/v

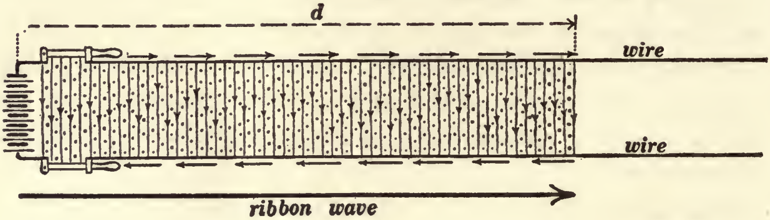

Fig. 101.

Showing the ribbon wave which shoots out from a battery which is suddenly connected to the end of a line.

seconds after the battery is connected. The short horizontal arrows represent current, the fine vertical lines represent the lines of force of the electric field, and dots represent magnetic lines of force as in Figs. 97 and 100.

Reflection.—For the sake of brevity and clearness we will discuss only the reflection of the ribbon wave, and such a wave will be represented by a single heavy horizontal arrow as in Fig. 101. When such a wave is turned back or reflected at the end of a line, the heavy arrow will be shown as turned back as in Figs. 102, 103 and 104. The voltage and current in the original wave are represented by E and I and the voltage and current in the reflected wave are represented by E_r and I_r as shown.

Reflection from open end of line.—The doubled arrow in Fig. 102 represents a wave which has been turned back or reflected at the open end of a transmission line. The necessary condition which must be satisfied at the open end of a line is that the actual current there be zero. Therefore we have:

$$I + I_r = 0 \qquad \text{(i)}$$

In addition to this we must have:

$$\frac{E}{I} = +\sqrt{\frac{L}{C}} \qquad \text{(ii)}$$

and

$$\frac{E_r}{I_r} = -\sqrt{\frac{L}{C}} \qquad \text{(iii)}$$

Now the voltage and current in the original wave are supposed to be given and the voltage and current in the original wave do, as a matter of course, satisfy equation (ii). From equation (i) we have:

$$I_r = -I \qquad \text{(iv)}$$

and from equations (ii) and (iii) we have:

$$E_r = +E \qquad \text{(v)}$$

Reflection at the open end of a line is therefore complete and it takes place with reversal of current.

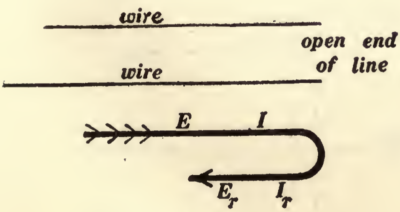

Fig. 102.

Showing a ribbon wave partly reflected from the open end of a line.

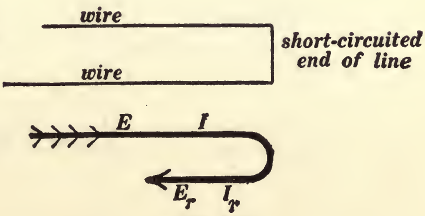

Fig. 103.

Showing a ribbon wave partly reflected from the short-circuited end of a line.

Reflection from short-circuited end of line.—The doubled arrow in Fig. 103 represents a wave which has been turned back or reflected from the short-circuited end of a line. The necessary condition which must be satisfied at the short-circuited end of a line is that the actual voltage across the end be zero. Therefore we have:

$$E + E_r = 0 \qquad \text{(vi)}$$

whence

$$E_r = -E \qquad \text{(vii)}$$

and, since equations (ii) and (iii) always apply, we get:

$$I_r = + I \qquad \text{(viii)}$$

Reflection at a short-circuited end of a transmission line is therefore complete and it takes place with reversal of voltage.

Transmission line oscillations which follow the switching on of a generator.—When a generator of negligible resistance and

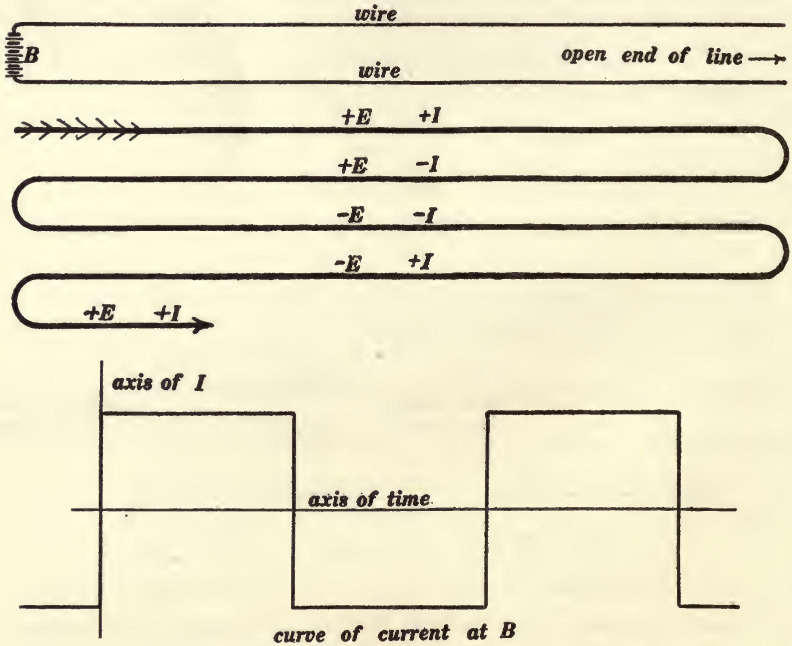

Fig. 104.

The upper part of the figure shows a ribbon wave which shoots out from a suddenly connected generator and is repeatedly reflected at both ends of the line. The lower part of the figure shows the value of the current at B as a function of elapsed time.

inductance is suddenly switched on to a line, a ribbon wave of generator voltage and corresponding current shoots out over the line. Assuming line resistance to be negligible and line insulation to be perfect, this ribbon wave is reflected back and forth as represented in Figs. 104 and 105, and by adding voltages and

currents in the successive laps of the ribbon wave a precise knowledge of the distribution of voltage and current over the line at any instant may be obtained. Thus, after a thousandth of a second, the total length of the ribbon wave would be 186

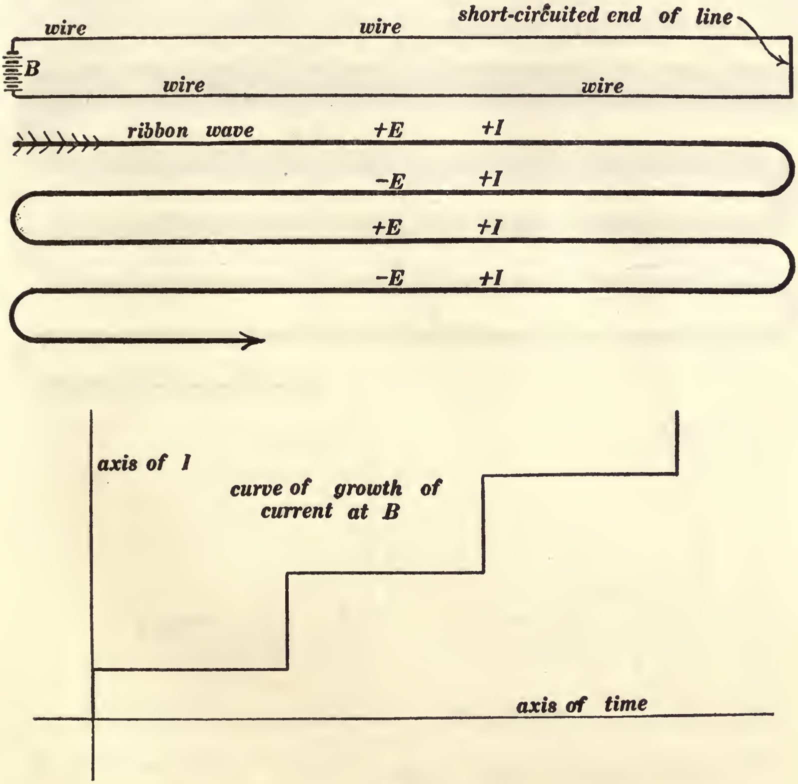

Fig. 105.

Same as Fig. 104 except that distant end of line is short-circuited.

miles which would give a definite number of complete laps and a fraction of the next lap. By adding a number of laps in this way, curves can be plotted showing the actual distribution of voltage and current over the line at any given instant; or values of current and voltage at a given point of the line can be found for successive instants and from this data curves can be plotted showing voltage or current at any point as functions of elapsed

times. The ampere-time curves in the lower parts of Figs. 104 and 105 were obtained in this way.

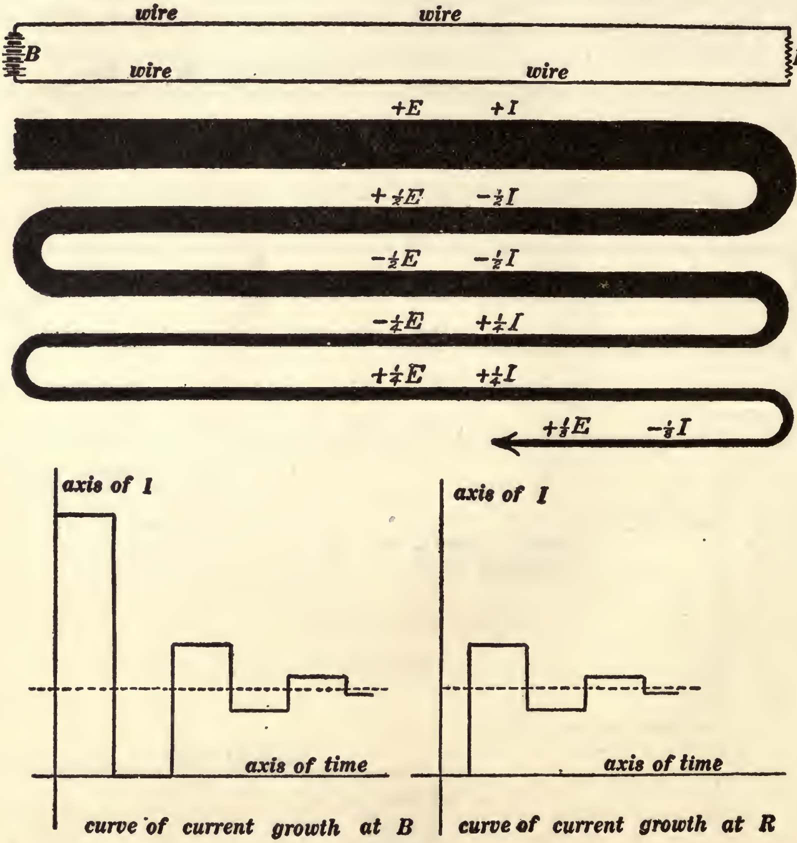

Fig. 106.

Same as Fig. 104 except that distant end of line is connected to a non-inductive circuit of which the resistance is $R = 3\sqrt{\frac{L}{C}}$. In this case the ribbon wave is only *partially* reflected from the end R and the successive laps of the ribbon wave are thereby greatly reduced in intensity as represented by the shading.

Note.—The character of the reflection at B in Figs. 104, 105 and 106 is determined by the condition that the sum of the voltages in the successive laps must always be equal to battery

voltage E, and of course there is always an odd number of laps at B, one, or three, or five, etc.

Note.—Figures 104, 105 and 106 show what takes place when a direct-current generator or battery is switched on to the line, *but on a line of moderate length a number of laps are formed in an excessively short interval of time, and therefore Figs.* 104, 105 *and* 106 *show quite accurately what takes place immediately after an alternator is switched on to the line, E being the value of the voltage of the alternator at the instant of closing the switch.*

Transmission line surges which are produced when a circuit breaker opens.—When a circuit breaker opens, the arc which is formed persists for a very long time, relative speaking, and the open gap in the circuit is filled with a fairly good conducting material which slowly loses its conductivity. It is about as nearly impossible to produce characteristic line surges by opening a circuit breaker as it would be to set up an abrupt water wave in a canal by allowing a cubic mile of soft mud to flow into the canal prism to stop a troublesome flow of water in the canal; and yet the moon, as a 60-cycle generator (60 cycles per month!), might produce a troublesome tidal wash in a large estuary while the attempt was being made to "open-circuit" the estuary in this Brobdignagian fashion! Let the reader consider this hydraulic analog carefully. It reproduces nearly all of the essentials of the electrical case. The conducting vapor in the arc of a circuit breaker is somewhat analogous to mud as a dam-building material.

Very little need be said of characteristic line surges in connection with opening of switches, except in the case of very long lines. When a line is short, or only moderately long, what takes place may be described quite accurately in terms of the simple ideas of the elementary alternating-current theory, where current values are supposed to rise and fall simultaneously throughout an entire circuit. The formation of a long arc between line wires in air and the quick snapping out of such an arc does, however,

produce an electric wave disturbance on a transmission line of moderate length, and the essential features of this case are shown in Fig. 107. Imagine the system to be short-circuited by an arc

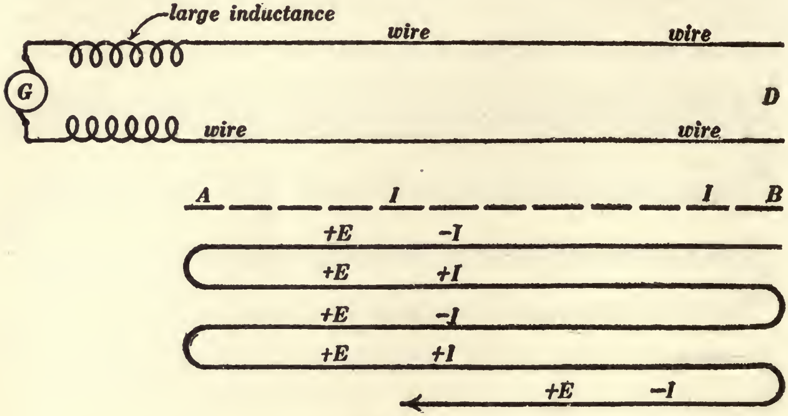

Fig. 107.

The dotted line AB symbolizes the initial current in the transmission line. The long heavy arrow with laps represents the ribbon wave which comes from D after the line at D is opened.

at the distant end D of the transmission line, voltage being reduced to a negligibly small value over the whole line, and a large current I being established in the line. The generator is to be thought of as having a large inductance so that the generator current cannot change perceptibly during the very short time required for the characteristic line surges. When the distant end of the line is opened, a ribbon wave shoots towards the generator, is completely reflected at the generator with reversal of current, again completely reflected at the opened end D of the line with reversal of current, and so on. The first lap of this ribbon wave wipes out the current in the line and lays down a certain voltage $E\,(=\sqrt{L/C}$ times the initial value of the current). The second lap of the ribbon wave lays down a double voltage, and the original current. The third lap wipes out the current again, and lays down a voltage equal to $3E$, and so on.

Note.—The character of the reflection at the generator in Fig.

107 is determined by the condition that the sum of the currents in the successive laps at A must always be equal to the initial current I because the current in the highly inductive generator does not have time to change perceptibly.

Note.—It must not be imagined that the sending out of a ribbon wave depends upon a continued supply of energy at the point where the ribbon wave originates. Thus in Fig. 107 the ribbon wave is superposed upon the initial current I, and the first lap of the ribbon wave wipes out this current. Therefore since the current is zero, there is no energy flow at all from the opened end. The second lap of the ribbon wave lays down the original current I and a doubled voltage $2E$, and this combination of voltage and current represents a flow of energy from the generator into the line.

THE TRADITIVE LAMP.

Long ago Bacon gave a list of the things needed for the Advancement of Learning, and, among other things, he mentioned A Traditive Lamp, or the Proper Method of Handing Down the Sciences to Posterity. This Lamp has not yet been discovered!

APPENDIX.

A VISITORS' LABORATORY OF PHYSICS.

The authors' experience shows that the average visitor who comes into the physical laboratory of a college or technical school is greatly interested in the simplest kind of a non-dazzling experiment if the experiment has a rational appeal and if this rational appeal is set forth in a simple explanation. Indeed the interest which the average visitor takes in genuine natural philosophy has led the authors to devote a great deal of time to visitors. A number of simple experiments, varied from time to time, are always set up, and every member of the department, including Clarence and Pete, has taken a share of the fun of edifying visitors.

This phase of the museum idea is certainly important, and the authors believe that our generously supported public institutions, especially, should have fully equipped *Visitors' Laboratories of Physics* and provide for the work of demonstration by members of the Physics Department staff.

The authors have tried all of the following experiments on visitors, some of them many times, and the greater portion of these experiments could be set up permanently in a space 25 feet × 80 feet with two or three small dark rooms adjoining, and the entire equipment need cost no more than four or five thousand dollars.

VISITORS' EXPERIMENTS.

5. Coin and card experiment.
17 and 18. The bicycle wheel as a gyroscope.
19. Curious gyroscope toy.
20. Sharp and blunt pointed tops and hard-boiled egg.
22. Model of Schlick balancer. Gyroscope oscillations.

24. Experiment with pivoted stool.
33. Gum camphor on water.
34. Experiments relating to oil flotation.
35. The sensitive flame.
36. The spit-ball experiment.
37. The water hammer.
38. The disk paradox and its water analog.
39. The jet pump and the atomizer.
40. Ball riding on air or steam jet.
41. Experiment on ship suction.
43. Curved flight of spinning ball.
44. High foul.
46. The Brownian motion.
50. The Trevelyan rocker.
53. Prince Rupert's drops.
54. The fire syringe.
55. Cloud formation experiment with large flask.
58. Crystallization of NH_4Cl and of $KClO_3$.
59. The boiling paradox.
63. Wood's metal spoon.
66. The recalescence of steel.
67. Retarded transformations. Hardening and tempering of steel. Exhibit sample of partly crystallized candy.
70. Thermo-elastic properties of rubber.
71. Large electro-magnet; the magnet should have pole pieces coming near together. (*a*) Show powerful attraction for a piece of iron; (*b*) Show eddy-current damping of motion of thick sheet of copper; (*c*) Show eddy-current damping of motion of closed copper ring; and (*d*) Show absence of eddy-current damping of motion of open-circuited copper ring.
84. Lead tree experiment.
88. Conductivity of glass at high temperatures.
91. Transformer experiment.
100. Electric dancers.

105. Electric oscillations maintained by an electric arc.
108. Rosin experiment.
113. The electric doubler.
115. The corona discharge.
116. Smoke deposition by Cottrell's method.
117. The ozonizer.
Discharge of electricity through gases. (*a*) Geissler tube discharge, (*b*) Crookes tube and cathode-ray shadow, (*c*) Magnetic deflection of cathode rays, (*d*) Heating effect of cathode rays, (*e*) Luminescence by cathode rays, (*f*) X-rays and fluoroscope, (*g*) Ionizing action of X-rays, (*h*) Ionizing action of alpha, beta and gamma rays, (*i*) The spinthariscope, and (*j*) Fog chamber for showing paths of alpha particles. See pages 139 and 140 of this volume.
118. Unit areas of sense of touch.
121. Looking at bright point under water.
Phantom boquet. A large concave mirror projects on top of an empty base a real image of a hidden boquet.
122. Visible beam beyond a large short-focus single lens.
123. Model of compound microscope.
124. Model of simple telescope.
125. Vision. Looking through a pin hole at a cord.
126. Vision. The coin box.
127. Vision. Seeing an object inverted when its image is erect.
128. Vision. Curious effect of two-eye vision.
129. Vision. Shadows of blood vessels on retina.
130. Vision. The stroboscope.
131. Vision. Reversal of sense of motion.
134. Astigmatism of simple lens of narrow aperture.
Looking at distorted drawings in cylindrical and conical mirrors. See catalogue of Max Kohl.
136. Chromatic aberration of the eye.
137. Projection of large image of sun by telescope used as a telephoto-lens.

138. Spectroscope demonstrations. (*a*) Continuous spectrum, (*b*) Bright-line spectrum, (*c*) Reversal of sodium lines, and (*d*) Dark-line spectrum (solar spectrum).

140. Spectroscopic analysis of light reflected from thin film. (*a*) Very thin soap film, (*b*) Moderately thick mica plate.

141. Looking at flaming arc through coarse diffraction grating.

143. Experiments on polarized light. *a*, *b*, *c*, *d*, *e*, *f* and *g*.

144. Experiments on color. *a*, *b*, *c*, *d* and *e*.

145. Reversal of order of sound sensations.

146. Musical sticks.

147. The Galton whistle. Determination of upper pitch limit of audibility.

148. Simple modes of vibration of string.

149. Simple modes of vibration of air column. Long glass tube whistle with lycopodium powder.

150. Chladin's figures.

151. Resonance. (*a*) With piano and voice, (*b*) With tuning fork and adjustable air column. A most striking experiment is to connect a telephone to an alternating current supply (the frequency must be about 150 cycles per second to give best results) and hold the telephone over the mouth of a tube which is slowly lowered in a jar of water.

152. Experiments on vowel sounds.

153. The action of the ear in the perception of tone color or timbre.

154. Beats.

155. Combination tone.

Note.—Details of experiments are described in main body of this volume (under given number) and in Franklin and MacNutt's *General Physics*.

FRANKLIN, MACNUTT AND CHARLES

Publishers of Educational Books

That our interest in selling books is not narrowly commercial is evident from the wide departures from traditional points of view in every one of the twenty-six volumes. This number includes Nichols and Franklin's *Elements of Physics*, three volumes, and Franklin and Williamson's *Elements of Alternating Currents*. We are propagandists rather than commercialists, and as propagandists we address this prefatory statement to particular groups of men as follows :

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(*b*) **To those specially concerned with scientific and technical education**, we recommend the philosophy and the biting humor that is scattered throughout the *Calendar of Leading Experiments*.

(*c*) **To teachers of physics and to teachers of electrical engeneering.** The earlier books on these subjects are widely known, but *General Physics* (1916) and *Elements of Electrical Engineering* Vol. I. (1917) represent wide departures from the earlier books, and they are, we believe, great improvements thereon from a teaching point of view.

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(*e*) **To teachers in trade schools and night schools** we recommend *Elementary Statics* (No. 8) and *Elementary Electricity and Magnetism* (No. 9).

(*f*) **To students of the non-mathematical sciences.** Franklin, MacNutt and Charles take pride in the fact that what they stand for is quoted in William James' book on *Pragmatism* as an extreme example of pragmatic philosophy. The references which are brought together on page 8 of this circular constitute a very simple and highly pragmatic treatise on the philosophy of the mathematical sciences.

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No. 1

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This textbook, with its brevity and clearness, its directness of treatment and its freedom from tedious repetition of the commonplaces of High School Physics, was intended to do away with the usual excessive amount of classroom coaching which is the despair of every physics teacher who aims to develop the power of analytical thinking and who tries therefore to hold his students accountable for mathematical ideas, mathematical formulations (not mathematical formulas, please note) and numerical calculations. And the use of the book in the classroom seems to show that it does in some measure accomplish what was intended. The students do read it, it tends to focus their attention on fundamentals, it helps to an unprecedented extent (in the authors' experience) to form precise ideas, and it enables the student to devour numerical problems.

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BY

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Published and for sale by

FRANKLIN, MacNUTT AND CHARLES

South Bethlehem, Pa.

Price $2.50 post paid

This book has to do primarily with lecture demonstrations in physics. The authors believe that High School Physics would be greatly helped by the use of more class-room demonstrations, they believe that such demonstrations can be provided for at a very small expense, and they consider that many of the demonstrations described in this book would be usable in the High School. The book has an Appendix on A Visitors' Laboratory of Physics, and certainly our State Colleges and High Schools could do a great work by developing this phase of the museum idea.

Secondarily this book sets forth the possibilities of an extended course in elementary dynamics, including the dynamics of wave motion; and from beginning to end the book is filled with semi-humorous reflections on the problems of the teacher of physics.

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"It will teach many a mathematician many a useful thing if only the mathematicians will take heed unto it." Professor E. B. Wilson, Massachusetts Institute of Technology.

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5. **Mechanics and Heat.** The Macmillan Co., 1910, " 1.75
6. **Elements of Electricity and Magnetism.** The Macmillan Co., 1909 . . . . " 1.60
7. **Light and Sound.** The Macmillan Co., 1909 . . . " 1.60

   NOTE—Books 5, 6 and 7 constitute a fairly complete treatise on the elements of physics.

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   NOTE—This book is in pamphlet form.

9. **Elementary Electricity and Magnetism.** The Macmillan Co., 1914 . . . . " 1.25

   NOTE—This book is suitable for trade schools and night schools.

10. **Advanced Electricity and Magnetism.** The Macmillan Co., 1915 . . . . " 2.00

   NOTE—Every student of electrical engineering needs something beyond the bare elements of electricity and magnetism, and this book is intended to supply this need. Contents are as fol'ows: Pages 1–74 summary of elements, pages 75–103 magnetism of iron; pages 104–120 ship's magnetism and the compensation of the compass; pages 121–164 electrostatics; pages 165–192 the theory of potential; pages 193–273 electric waves; and pages 274–297 the electron theory.

Practical Physics. By Franklin, Crawford and MacNutt. The Macmillan Co., 1903. A Laboratory Manual of Physics in three volumes.

11. **Volume I.** Precise Measurements. Mechanics and Heat . . . . " 1.25
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16. Vol. II. Alternating Currents 1908........ " 3.50

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W. S. Franklin.

17. Vol. I. D. C. and A. C. Machines and Systems 1917.......................... " 4.50

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Note. A Key to Problems is supplied with teachers' desk copies of these two books.

DYNAMO LABORATORY MANUAL. Franklin and Esty.

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This volume contains the portions of Franklin and Esty's *Elements* which relate to D. C. and A. C. machines.

22. ELECTRIC WAVES. W. S. Franklin, 1909.. " 3.00

The more elementary portions of this volume are given in a carefully revised form on pages 193–273 of Franklin and MacNutt's *Advanced Electricity and Magnetism*, book No. 10 in the above list.

Many biologists and many purely experimental chemists would like to have a clear insight into the methods of the mathematical sciences; but every science is now presented as for the narrow specialist, or for the dilettante, and the student must choose everything in a particular field, or nothing. An exclusive interest in a particular science seems to be the only interest that is recognized as serious.

The following references bring together a simple and highly pragmatic treatise on

THE PHILOSOPHY OF THE MATHEMATICAL SCIENCES

and with the help of this simple treatise, which it is hoped some time to publish separately, any mature student can get a clear understanding of the methods of the mathematical sciences with a very moderate expenditure of time and effort.

The infinitesimal calculus. See pages 593–597 of book No. 1 for a discussion of the meaning of the infinitesimal calculus, and see chapter one of book No. 3 (40 pages) for a precise and clearly intelligible survey of differential and integral calculus. This chapter is presented in carefully revised form in the Supplement which accompanies the book.

Methods in physics. The four fundamental methods in physics are (*a*) The method of mechanics, (*b*) The method of thermodynamics, (*c*) The method of atomics (the atomic theory), and (*d*) The method of statistics.

The philosophy of elementary mechanics is discussed at some length in book No. 2. See pages 119–123 of book No. 1 for a discussion of the atomic theory in contrast with thermodynamics; see pages 322–325 of book No. 1 for a discussion of the method of mechanics as contrasted with the method of atomics and see a very simple and suggestive discussion of the use of the statistical method in physics in *Science*, pages 158–162, August 4, 1916.

The conservation of energy and the law of entropy. Any scientific work, to be fruitful, must be to some extent conditioned by the great generalizations of physics; but these generalizations need to be tempered, in their wide acceptance, by a full appreciation of the fact that they are very largely of postulate content. The great value of precise ideas (mathematical ideas) is that they open the mind for the perception of the simplest evidences of a subject, and mathematical thinking is a necessity *in the mathematical sciences;* but any kind of an idea unconditionally held closes the mind completely to contrary evidences. This matter is discussed somewhat naïvely on pages 75–90 and 99–101 of book No. 4.

The law of entropy is, perhaps, rigorously true for a world in which no organizing agencies are at work coördinating things which are apart in time and apart in space, but living forms seem to be subject to the law of entropy only in that the easiest line of delevopment of an organism which must use free energy is the line of development in which an excessive burden of coördination is not necessary at the beginning.

The conservation of energy is discussed on pages 60–70 and 119–125 of book No. 1.

The law of entropy (the second law of thermadynamics) is discussed on pages 153–168 of book No. 1.

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